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ABSTRACT

The purpose of this document is to provide a framework around which systems and schools may build their mathematics curriculum. Important components of a mathematics education for the majority of students are identified. This document is presented in two parts. Part I, Principle for School Mathematics, addresses questions such as what is mathematics, why is it important and for whom, what are the goals of school mathematics, and what conditions will support effective learning of mathematics. Part II, the Scope of the Mathematics Curriculum, describes mathematical understandings, skills, knowledge, and processes which should typically by made available to students. These are categorized as attitudes and appreciations, mathematical inquiry, choosing and using mathematics, space, number, measurement, chance and data, and algebra. Contains 39 references. (ASK)

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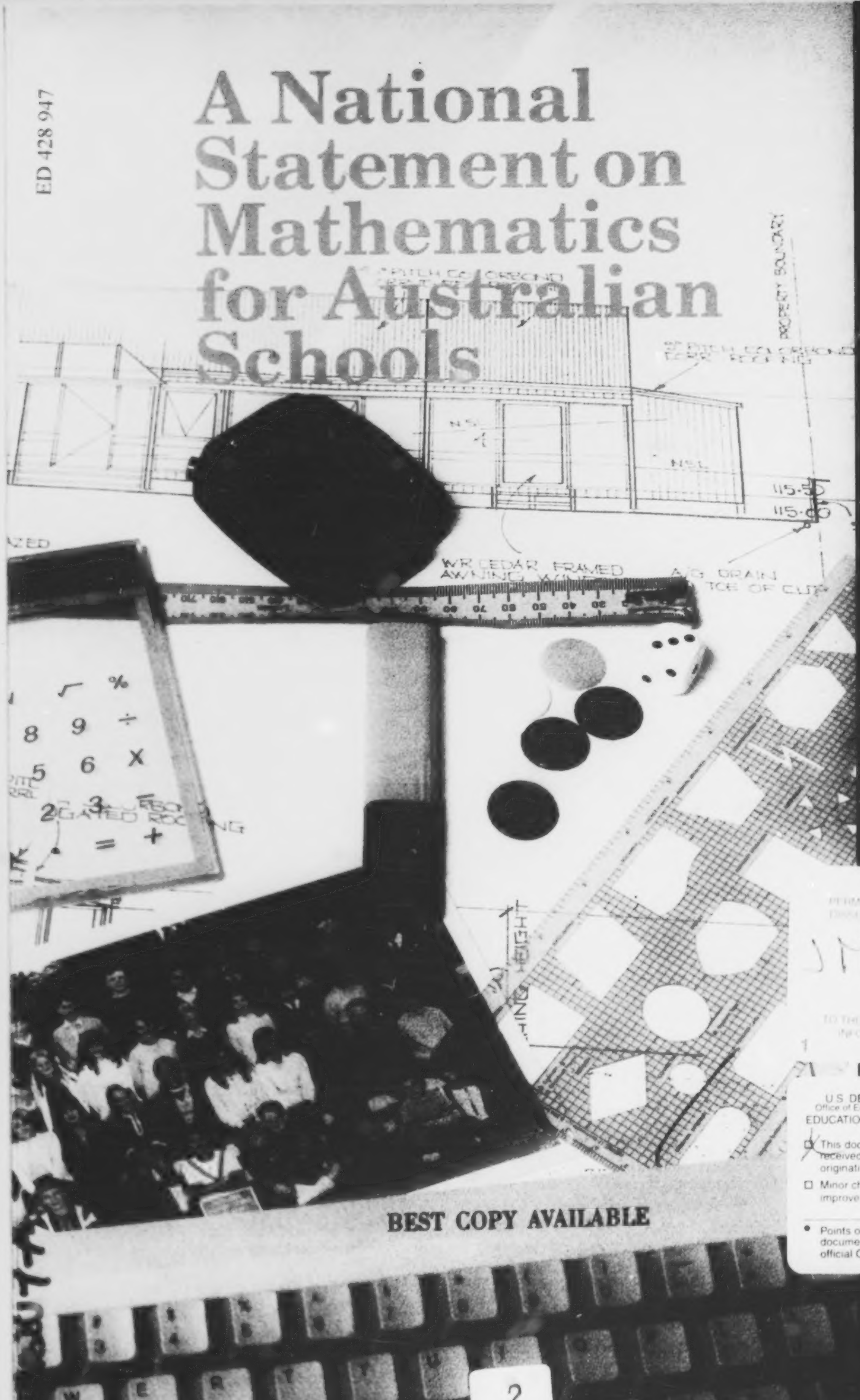

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A National Statement on Mathematics for Australian Schools

A joint project of the States, Territories and the Commonwealth of Australia initiated by the Australian Education Council

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Foreword

Students presently in Australian schools will spend most of their lives in the twenty-first century. It is difficult to know exactly what kind of education they will need in order to function effectively as consumers, community members and workers. Changes, however, will be needed.

Mathematics pervades all aspects of our lives, whether at home, in civic life or in the workplace. Mathematics has been central to nearly all major scientific and technological advances. Also, many of the developments and decisions made in industry and commerce, the provision of social and community services, and government policy and planning, rely to an extent on the use of mathematics. The rapid spread of cheap and powerful calculators and computers has dramatically extended the capacity of people, both individually and collectively, to use and generate information as a basis for decision making. Without the capacity to make informed judgements, the user may become a servant of technology, with possibly detrimental consequences. The critical and productive application of this technology will make greater demands on the user's mathematical knowledge.

We need to aim for improvement in both access and success in mathematics for all Australians. All Australians must leave school well prepared to meet the demands of their future lives and with the knowledge and attitudes needed to become lifelong learners of mathematics.

In April 1989 the State, Territory and Commonwealth Ministers of Education endorsed ten common and agreed national goals for schooling in Australia. As a result, the States and Territories have entered into the collaborative development of statements about the curriculum of Australian schools. This is the first such statement. Statements for other areas of the curriculum will follow.

When commissioning the development of the *National Statement on Mathematics for Australian Schools* the Australian Education Council determined that it should describe the mathematical understandings, knowledge and skills to which students will typically be exposed and teaching methods which are likely to encourage productive learning strategies and positive attitudes towards their involvement in mathematics.

The purpose of the *National Statement on Mathematics for Australian Schools* is to provide a framework around which systems and schools may build their mathematics curriculum. It does not provide a syllabus or curriculum and, indeed, its structure makes it inappropriate for direct use in that way. Rather, it should provide a foundation for appropriate courses which will meet students' needs and reflect advances in our knowledge — both of the subject mathematics itself and of the ways in which students learn mathematics. The *Statement* encourages innovation and experimentation so that all learners have a positive experience of mathematics.

The project was managed by a Steering Committee representing the Directors of Curriculum from State and Territory Government education systems and the Commonwealth. The Steering Committee set up a Project Team to undertake the writing of the document and a Reference Group of mathematics curriculum officers from each State and Territory. As the writers carried out their task they took advice from consultants, tertiary mathematicians and mathematics teacher educators, and continually referred all material to the Reference Group. Throughout the writing process, formal and informal consultations occurred at both State/Territory and national levels. Drafts of this statement were circulated for discussion and comment to people drawn from different sections of the community such as parents, teachers (from both government and non-government sectors), teacher educators, professional associations, mathematicians, curriculum developers, community groups, employers and unions.

This statement is intended essentially for use by educationalists and the teaching profession; a companion statement has been prepared for the broader community. We commend this statement, and the community statement, *Mathematics in our Schools*, to educators, parents, mathematicians, employers and members of the general community.

Australian Education Council
December 1990

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Introduction

The *National Statement on Mathematics for Australian Schools* is the result of a collaborative project by the States, Territories and Commonwealth of Australia. It documents areas of agreement between education systems about directions in school mathematics, the principles which should inform curriculum development and the extent and range of school mathematics.

The purpose of the Statement

The purpose of the *Statement* is to provide a framework around which systems and schools may build their mathematics curriculum. It identifies important components of a mathematics education for the majority of students. It is descriptive rather than prescriptive. It does not provide a syllabus or curriculum and, indeed, its structure makes it inappropriate for direct use in that way.

Precisely how it will be used, and by whom, will depend upon the particular distribution of responsibilities for curriculum development within each State and Territory. In some places, it will be used by education authorities to review the advice and curriculum support they currently provide for the teaching of mathematics. In other places, schools may use the document to assist them in revising their mathematics curriculum. Teacher education institutions will also find it of use in planning curriculum courses. Thus, its major audience is curriculum developers at the system, regional or school level, but it will also be of interest to members of the broader community.

Education benefits from diversity. Accordingly, the *Statement* encourages innovation and experimentation, the development of learning experiences appropriate for particular children in particular schools, and local community involvement in the education of their children. It should not be interpreted as placing limits or restrictions on the scope of the mathematics curriculum or on the range of pedagogies adopted.

The present document is intended to represent one phase in an ongoing process of collaboration between States, Territories and the Commonwealth. As such, the document itself is to be subject to periodic revision. The terms of reference for the preparation of this statement require that areas for future collaboration be identified in order to support the directions identified in this phase. Areas which have been identified include: broadening the range of assessment strategies and tasks available for mathematics, classroom resources to support directions suggested within this *Statement*, further work on the content and structure of mathematics during the post-compulsory years (including articulation with further education), the implications of new technologies for mathematics in schools, and the professional development of teachers to assist them to implement the recommendations of the *Statement*.

Overview of the Statement

School mathematics is changing. There are several reasons for this. There has been a rapid growth of mathematics and new technologies are influencing how mathematics is produced and applied. The mathematical demands of daily life, civic responsibilities and work are increasing and the nature of the mathematics used is changing. In the interests of social justice and the maintenance of Australia's health, and environmental and economic well-being, we need to broaden access to and success in mathematics. Finally, we know much more about how students best learn mathematics and of the implications for classroom practice and assessment.

The question of what students will need in a rapidly changing world is one that challenges all educators and, in particular, mathematics educators. It is impossible to know exactly what mathematical concepts, skills and processes people will need during their lives in order to function as consumers, community members and workers. Furthermore, quite different mathematical skills may be needed by individuals appearing to have quite similar lifestyles or forms of employment.

This document recognises that mathematics curricula in Australia must respond to changing circumstances, that the mathematical demands on people will vary throughout their lives, and that schooling cannot prepare people for all the mathematics they are likely to need in their civic and working lives. If people are to continue to use mathematics, they must develop the competence, confidence and interest needed to become lifelong learners of mathematics.

The *National Statement on Mathematics for Australian Schools* is presented in two parts. Part I, Principles for School Mathematics, addresses the following questions:

What is mathematics, why is it important and for whom?
What are the goals of school mathematics?
What conditions will support effective learning of mathematics?

Part II, The Scope of the Mathematics Curriculum, describes mathematical understandings, skills, knowledge and processes which should typically be made available to students. These are described under eight headings:

Attitudes and appreciations
Mathematical inquiry
Choosing and using mathematics
Space
Number
Measurement
Chance and data
Algebra

These have been used as a means for structuring the document; they are not intended to provide a structure for teaching mathematics. There are significant overlaps and inter-connections between the sections which should be considered when undertaking detailed curriculum planning. The unity of mathematics should be emphasised throughout and students should be helped to draw upon the whole range of their mathematical experiences in the solution of their problems.

Principles

for school mathematics

The importance of mathematics

What is mathematics?

Mathematics involves observing, representing, and investigating patterns and relationships in social and physical phenomena and between mathematical objects themselves:

> **Mathematics is often defined as the science of space and number ... [but] a more apt definition [is that] mathematics is the science of patterns. The mathematician seeks patterns in number, in space, in science, in computers, and in imagination. Mathematical theories explain the relations among patterns ... Applications of mathematics use these patterns to 'explain' and predict natural phenomena ...**[1]

Mathematical knowledge is not empirical knowledge in that its source is not physical reality; rather its source is patterns and relationships created in the mind.[2] The physical properties of apples (colour, mass) are *in* the apples and can be known through observation. However, when we order the apples according to size, the order is not in the apples but rather in the relationships we create between the apples. The colour of the apples is an example of physical knowledge; their order is an example of 'logico-mathematical' knowledge,[3] as are the number of apples and the differences or similarities between them.

In this sense, mathematics always involves abstraction.[4] We observe similarities between objects or events in the physical or social world, or between mathematical objects such as numbers. Having observed similarity, we represent it in some way using words, symbols or diagrams. For example, the mathematical object 'rectangle' is derived from such things as rock faces, the vertical section of trees and doorways. 'Five' is abstracted from all the collections which can be matched with the fingers of one hand, while the idea of 'prime number' is derived from other mathematical objects (that is, from certain numbers). The general expression $y = mx + b$, or a straight line graph, each represents relationships observed between certain pairs of quantities (for example, the amount of grapes purchased and the total cost, the distance travelled and the time taken, or the length of the side of a square and its perimeter).

Mathematics provides powerful, precise and concise methods of representing patterns and relationships. These mathematical representations are then treated as objects in their own right. They can be manipulated according to specific rules of logic either for a particular purpose or in an exploratory fashion, the intention being to find new relationships between the expressions.[5] For example, beginning with an idea of 'rectangle', we represent its essential features in a definition. From this we can conjecture about other relationships about rectangles and attempt to justify or refute our hypotheses.

Through cycles of conjecture, justification and refutation we may derive relationships such as: 'a four-sided figure is a rectangle if and only if its diagonals are equal in length and bisect each other'. We can then apply this general rule to any object which can be thought of as rectangular. Thus, we can ensure that a doorway is rectangular by ensuring that its diagonals are the same length and bisect each other. We do not have to establish that the relationship holds in the special case of doorways.

These processes of observation and investigation of pattern, abstraction, representation, generalisation, analysis and justification may thus result in statements of broad applicability. Indeed, one of the aims of mathematics is to identify within each field a small number of ideas and properties from which other properties can be derived.[6] The point is that the derived properties can be applied in any context in which the original small number of ideas and properties apply.

A creative activity, mathematical investigation involves invention, intuition and exploration. At times, the problems investigated are generated within the subject field of mathematics and thus relate directly to mathematical abstractions. In such cases, there is no immediate interest in whether the relationships derived relate to any social or physical phenomena. The purpose is explicitly mathematical.

At other times, the problems are generated from the 'real' world. In such cases, they need to be disembedded from their particular context, the essential relationships within the context represented mathematically and the problem stated in mathematical terms. Mathematical techniques may then be applied to the problem as mathematically stated.

Results obtained in either of these ways may influence each other. For example, the development of differential geometry by Riemann, Minkowski, and others, gave Einstein the tools he needed to formulate mathematically his theory of general relativity. On the other hand, the search for mathematical descriptions of shapes such as coastlines and ferns, or of phenomena such as turbulent flow, has led to active mathematical theories of non-linear dynamics and fractal geometries.

Why is mathematics important?

Mathematics is an integral part of a general education. It can enhance our understanding of our world and the quality of our participation in society. It is valuable to people individually and collectively, providing important tools which can be used at the personal, civic and vocational levels.

Mathematical ideas about number, space, movement, arrangement and chance — hundreds, sometimes even thousands, of years old — are used in everyday life by the majority of people. They are also used in the most modern mathematics, science, economics and design. This durability and versatility of many mathematical ideas leads some people to view the subject as unchanging, even though mathematics is rapidly growing. The growth results from new ideas or notions. Some has been a result of the use of sophisticated computing technology and some mathematicians argue that this technology is driving a new revolution in mathematics:

> **The rapid growth of computing and applications has helped cross-fertilise the mathematical sciences, yielding an unprecedented abundance of new methods, theories, and models ...**

> **Computers change not so much the nature of the discipline as its scale: computers are to mathematics what telescopes and microscopes are to science. They have increased by a millionfold the portfolio of patterns investigated ... As this portfolio grows, so do the applications of mathematics ...[7]**

The raising of levels of confidence and competence in mathematics is essential for widespread scientific literacy and for the development of a more technologically skilled workforce. The former is necessary for the 'personal competence, social cohesion, employment prospects and the free flow of comprehensive information that makes democracy workable';[8] the latter for the economic competitiveness without which we will lose the basic economic preconditions for a democracy.

Mathematics is used in daily living

The most obvious daily uses of mathematics involve number and quantification. In caring for our homes and in making purchases, we count, measure and carry out simple computations. We also use spatial and measurement skills to read maps or house plans, and to judge how much paint is needed to finish the decorating. Understanding weather bulletins and economic indicators requires a basic understanding of chance and statistical inference. Those same understandings are involved when we analyse benefits and risks in choosing between loan repayment schedules or insurance plans.

Mathematics is used in civic life

A democracy assumes the capacity of all members of the community to participate in the public debates and social action through which important decisions are made. Increasingly, fully informed participation requires mathematics.

Modern communication technologies have meant that people are confronted with much more information than was previously the case. Those who wish to remain informed about many important issues of our time are forced to deal with masses of data which may even be conflicting. To make reasonable sense of the evening news bulletin requires mathematical understandings about data collection and handling, and statistical inference, which go considerably beyond those required even twenty years ago. In the health and environmental arenas there are almost always trade-offs between quality and cost[9] (e.g. of medicines or screening tests for contaminates or diseases). Making informed choices will often require a general understanding of the mathematics underlying the analysis of costs and risks.

Arguments which are couched in mathematical terms will often intimidate and mystify people. Lyn Arthur Steen[10] suggests that 'a public afraid or unable to reason with figures is unable to discriminate between rational and reckless claims in the technological arena'. Those who do not understand mathematics, or who believe they do not understand mathematics, may be susceptible to economic, social and political manipulation.

Mathematics is used at work

Mathematics is used widely in the workplace, although users may not always recognise the extent of its use. Checking the daily takings, organising the week's payroll and budgeting for a sales campaign make clear and widely recognised demands.

Other uses vary from reading tables and specifications to recording information systematically, culminating in the use of sophisticated inventory control systems. Following a blueprint, deciding how much paper is needed for a sales brochure, and scaling and arranging pattern pieces to minimise the material used, all make mathematical demands of workers. Making measurements, calculating averages and the variability of measures, and comparing these with acceptable tolerances, are important parts of quality control systems.

The physical sciences, engineering and computer science have traditionally been regarded as requiring a high level of mathematics. Fields such as geography, biology, art, economics, fashion design and management, however, increasingly use mathematical techniques. Mathematics underpins most industry, trade and commerce, social and economic planning, and communication systems.

Mathematics is a part of the culture of all Australians

Understanding ourselves requires that we understand how mathematics is integral to our ways of thinking about the world. Playing and explaining, along with counting, measuring, locating and designing have been identified as probably universal in the development of mathematical ideas in different cultures.[11] However, while mathematical thinking is part of the cultures of origin of all Australians, we may not all share the same basic conceptions. For example, questions such as 'how many' and 'how much' are central to the value system of European Australian society. On the other hand, in many traditional Aboriginal communities the important questions were 'who' and 'whose'.[12] In these communities the combinatoric thinking needed for describing complex kinship patterns was much more important than the idea of quantifying aspects of reality, which underlies a great deal of Western mathematics. To learn the mathematics of another culture is, to an extent, to become bicultural.

Our ways of thinking influence, and are influenced by, mathematics. They change over time and are unlikely to be the same for all people. Nonetheless, an international 'body of knowledge' of mathematics has resulted from the convergence of mathematical activities of many cultures, both past and present. Like music, art, literature and sport, it has its own patterns, its repetitions and rhythms and associations with ritual and leisure. To an extent, mathematics also has a life of its own which transcends many cultural differences.

For whom is mathematics important?

There is considerable anecdotal and research evidence to suggest that many people dislike mathematics and may even feel intimidated in situations in which it is used.[13] Of considerable concern is the effect on individuals of having to deal with an increasingly mathematically oriented society while feeling inadequate or alienated from mathematics.

The dearth of people willing to participate in mathematics-related activities is also likely to become a significant labour market problem in a technologically oriented society. It is predicted that demand in Australia for mathematically skilled people will rise but the supply fall. This is due both to Australia's population demographics and to shortages in appropriately skilled people across the world, which means that we can no longer expect immigration to fulfil our shortfall.[14]

We must, therefore, ensure that more students leave school enthused by, and justifiably confident with, mathematics. This implies that a wider range of students should be expected and encouraged to participate in mathematics than has previously been the case.

A wider range of people should gain access to and success in mathematics than has been the case in the past

It is a premise of this *Statement* that all but a few exceptional students are capable of achieving the mathematical confidence and competence needed for personal and civic activities, the skills needed for vocational purposes, and some appreciation of the social and cultural significance of mathematics. For some students this will take longer than for others.

In the past, the tendency was for those students who were least successful in mathematics to stop studying it as early as possible. Many did not remain in formal education beyond the compulsory years. The mathematics courses available to those who did remain at school were often designed to be preparatory for higher education rather than immediately or personally useful. Consequently, many students saw little value in continuing to develop their mathematical understanding and did not choose to take more mathematics. In particular, those who were mathematically least prepared by the end of their compulsory schooling were also the least likely to learn more mathematics at school or in further education.

This was regarded as reasonably satisfactory. Many people used very little mathematics beyond what they learned during their primary schooling, or otherwise learned quite job-specific mathematical techniques. Full employment was available to people who often had not much more than primary level mathematics, and sufficient people were moving into mathematically related fields.

That situation no longer exists. As suggested earlier, the mathematical demands of daily, civic and working life are increasing. Job reclassifications are breaking down barriers which previously confined people to narrowly defined categories of work. Mathematics will be critical for people who wish to avail themselves of the advantages of job restructuring. If people are to cope with these changes and to benefit from the opportunities to work in a range of positions, then their early learning will need to prepare them to be learners of mathematics for the rest of their lives.

The number of students completing a full secondary education is increasing rapidly. This provides an increased opportunity to ensure that many more students leave school confident and competent to meet the mathematical demands of their future lives.

Access to and success in school mathematics should be independent of gender, social class or ethnicity

Participation in mathematics in Australia has been too dependent on being a member of particular groups in society. The under-participation of girls in mathematics has for some time been acknowledged as a concern. Under-achievement by many working-class and Aboriginal children and by some children from non-English speaking backgrounds is also now well documented.

Children and adolescents will vary in personal interest in mathematics and the extent to which they value it. When these differences occur along gender, social class or ethnic lines, the implication is that the mathematics curriculum is not equitable in meeting the needs of different groupings of the community. Whether a particular student gains the full benefit from mathematics may be influenced by a range of personal characteristics and circumstances. It will also depend on the qualities of the mathematics offered.

We are now beginning to understand some of our past curriculum practices in mathematics which have disadvantaged groups of students. For example, many of the contexts in which mathematical concepts were developed, applied and assessed were more likely to be central in the lives of boys than in the lives of girls. In some subtle and other not-so-subtle ways, the message was communicated to girls and boys that mathematics was of more relevance in the lives of adult men than adult women.[15] In a similar way, the mathematics curriculum has tended to emphasise values and concerns which are more middle class than working class, and to draw on experiences which are more relevant to children of Anglo-Celtic descent than those of Aboriginal descent or those from non-English speaking backgrounds.[16]

The complex interaction of linguistic and cultural factors on the learning of mathematics is only beginning to be understood.[17] For example, some traditional Aboriginal communities have ways of organising the world which are not based on counting and quantifying. Children from these communities may well enter school with little number experience and bring little meaning to the tasks of counting and quantifying. They are likely, however, to have much more highly developed spatial skills than other children, since these are vital for effective functioning within their environment and are therefore highly regarded and in constant use.[18] This has two implications. Firstly, the learning experiences which are provided should build on the strengths with which students come to the classroom. Secondly, we should avoid interpretations of 'ability' or 'intelligence' based on culturally narrow interpretations of important knowledge.

There has been considerable research into mathematics in traditional Aboriginal communities and this has had some influence on the teaching of mathematics in these communities. Most Aboriginal Australians, however, live in rural and urban areas, but there is little research available on the linguistic and cultural influences on their learning of mathematics.

Surface facility with English can often mask the language difficulties of students for whom English is a second language or with non-English speaking backgrounds. Sometimes such students are regarded as lacking in mathematical ability when they are actually experiencing problems with the formal language of the mathematics classroom.[19]

In order to provide education in mathematics which is inclusive of all Australian students, we will need to acknowledge the diversity of background of students in the range of activities which is offered and the expectations which are made of students. Ensuring that acknowledging diversity does not become stereotyping is not always an easy matter. School mathematics should build on what students already know, but it should also broaden students' horizons and the range of contexts in which they can function.

Access to and success in mathematics is important for students with special needs

Although the basic mathematical concepts and processes are the same for all students, each class will contain some students who are special in some way.

Some will have particular interests and talents in mathematics. We should encourage enthusiasm and show that we value the development of mathematical capabilities. We should also be vigilant to ensure that mathematical interest and talent are able to flourish regardless of geographic location, gender, social class or ethnic origins.

Other students may have an intellectual, physical or sensory handicap which interferes in some way with their capacity to engage in all the experiences of typical classrooms. A variety of different curriculum provisions and teaching strategies will be needed in order to accommodate their special needs. Whatever their particular needs or abilities, *all* students have the right to learn mathematics in a way that is personally challenging and stretches their capabilities. Achievable and satisfying tasks are an important prerequisite for success. Technology has already become a powerful educational tool for many special students, enabling access to education and careers not previously possible.[20] We need to build on these developments.

Goals for school mathematics

All Australian students will include mathematics in their studies during the compulsory years of schooling. Increasingly they are also encouraged to study it during the post-compulsory years. In some Australian States and Territories some study of mathematics is now required in the post-compulsory years. This reflects the importance attributed to mathematics across Australia.

Increasing the amount of mathematics studied by Australian students is important. More important, however, is ensuring that the mathematics which students do is rewarding for them individually and for their communities.

Students should develop confidence and competence in dealing with commonly occurring situations

An important goal of mathematics education is for students to develop the capacity to deal with commonly occurring or familiar mathematical situations readily and almost automatically. Everyday decision-making situations often involve us in asking questions about the cheapest, best, biggest, quickest or most likely. Many such questions require facility with number and measurement. For example, in order to choose the correct size packet of grass seed the home gardener will probably make some rough measurements and estimate the area of ground to be covered. This will require a facility with the units of measurement used on the pack. At the supermarket, the shopper may add and subtract amounts of money mentally, or 'round' money or measures and multiply or divide by one-digit numbers. In taking out a loan, the sensible shopper would study details of repayment schedules and total interest bills or use a scientific calculator in order to compare loan packages.

In each case, selecting the information needed in order to make a decision, and obtaining and interpreting that information, make quite considerable demands on numeracy. Since repetitive calculations are now routinely performed by relatively inexpensive machines, few such tasks require speed and accuracy with laborious 'paper-and-pencil' computation. Using calculators and computers productively does, however, require a greater depth of understanding of mathematical ideas than is often realised.

The question of which standard applications warrant specific attention in school mathematics is one which vexes educators and for which there is unlikely to be complete consensus. Some standard applications are in common use by the majority of people but the number of these is not as great as is sometimes supposed. Procedures which were efficient and important in their own right in pre-calculator days may now be less so. Many standard applications will be used frequently by particular groups within the community and almost never by other groups. Some may be used regularly during certain periods of an individual's life and rarely thereafter. On the basis of an analysis of the mathematical demands of certain trades, Foyster[21] comments:

> **... a given worker will, at a particular stage in his or her working life, use skills which are easy to define, but they will be different from skills used at other stages of the same worker's career, and may differ depending on just which area of the given trade the worker is operating in.**

Procedures which are very practical in some contexts may be less so in other contexts. For example, while most people may rarely need to find the volume of an irregular solid, a builder may do so frequently in order to estimate the cement needed for foundations. The builder, however, is likely to use a procedure which is quite specific to the constraints of measuring a building site.

Given the impossibility of identifying precisely which skills particular students will need in the future, narrow vocational views of mathematics are likely to be unproductive. Rather better approaches will be those which emphasise the development of positive attitudes towards their involvement in mathematics, problem solving and applications, working systematically and logically, and communicating with and about mathematics.

Students should develop positive attitudes towards their involvement in mathematics

The development in students of confidence about their capacity to deal with mathematical situations must be a high priority. The Cockcroft report of the inquiry into the teaching of mathematics in England and Wales[22] argued:

> **Most important of all is the need to have sufficient confidence to make effective use of whatever mathematical skills and understanding is possessed, whether it be little or much.**

We must increase the number of students who are enthused by and successful with mathematics and who wish to remain involved in it for personal, civic or vocational reasons. Therefore, mathematics curricula should explicitly address the development in students of positive attitudes towards mathematics and towards their continued involvement in mathematics.

All students, at all levels of schooling, should have opportunities to experience the excitement and pleasure that mathematical investigations can bring and to apply mathematics to problems which are personally interesting or which they regard as important. In the post-compulsory years, mathematics courses should reflect the wide range of contexts in which students may need to function in the future.

Students should develop their capacity to use mathematics in solving problems individually and collaboratively

A challenge for education is to develop the attitudes and knowledge which will enable students to handle familiar tasks easily and efficiently and also to deal with new or unfamiliar tasks.

Students should gain considerable experience in dealing with non-routine mathematical problems and unfamiliar situations. Choosing and using mathematical ideas to understand, to explain and to solve, can and should happen at every level of mathematical development. Considering whether mathematics might help to deal with situations which are not necessarily mathematical and judging whether a situation is one in which mathematics might appropriately be used are important. Students need to recognise when mathematics might be useful, choose the mathematics, do the mathematics, and evaluate its effectiveness in the circumstances.[23]

Facility with certain techniques can strengthen our capacity to inquire and to explore unfamiliar situations involving mathematics. Being able to perform certain procedures automatically frees us to think about the larger task at hand.[24] Being able to perform them reliably reduces the risk that small errors will prevent us from observing general patterns and relationships. Therefore, practising techniques such as those involved in mental arithmetic or in rearranging algebraic expressions can contribute to problem solving and should be a part of mathematics.

If such techniques are developed and practised in isolation, however, students may not recognise their power and usefulness. Also, too great an emphasis on predictable mathematical tasks may undermine the capacity of students to deal with unfamiliar tasks.

A second challenge for education is to develop an appreciation of both individual and collaborative activity and the skills needed in order to work productively in each of these modes. Often regarded as a rather solitary activity, mathematics develops through the interaction of communities of people working mathematically. Furthermore, the posing of problems and their solution almost always involve people in working together. At times, people will bring different skills to the task and take different responsibilities in developing solutions. At other times they will focus on the same task in order to to clarify and extend their own thinking. School mathematics should encourage and strengthen these ways of working.

Students should learn to communicate mathematically

A command of mathematical terminology and verbal, symbolic, diagrammatic and graphical modes of representation is essential in learning mathematics and is part of numeracy.

The process of developing and building up mathematical knowledge through describing, questioning, arguing, predicting and justifying almost always requires a sharing of ideas. The productive sharing of ideas depends upon the clarity with which one can express oneself. Mathematical communication skills are needed in order to understand, assess and convey ideas and arguments which involve mathematical concepts, or are presented in mathematical forms.[25] They are also needed in order to use text and other media to continue to learn mathematics.

In order to analyse and interpret certain material, students will need to understand the nature of mathematical arguments and the relationship between mathematics and its applications. For example, appreciating the nature of a mathematical model is necessary in order to make sense of some scientific and economic arguments. Some appreciation of the difference between an empirical and a logical argument and between a deterministic and a statistical argument is also important. An understanding of the general principles of model building and of different ways in which mathematics is used in reaching conclusions is an important part of informed numeracy.

Students should learn techniques and tools which reflect modern mathematics

Modern developments in the way mathematics is produced and applied raise questions about what basic mathematical ideas and perspectives students should acquire in school. For example, while school mathematics has tended to present a view of the world in which events are inevitable consequences that can be described by rules and formulae, there is an increasing emphasis on random models of the world. Also, we no longer assume that most natural and social phenomena can be thought of as more or less continuous (e.g. using calculus) and recent years have seen rapid growth in discrete methods (e.g. using networks).

Many of these changes are a result of the widespread use of computers. Paul Zorn[26] makes this point compellingly:

> **Computers do more than help solve old problems. They lead to new problems, new approaches to old problems, and new notions of what it means to solve problems. They change fundamental balances that have defined the discipline of mathematics and how it is pursued: continuous and discrete, exact and approximate, abstract and concrete, theoretical and empirical, contemplative and experimental. Computers change what we think possible, what we think worthwhile, and even what we think beautiful.**

Computers and calculators provide students with opportunities to investigate certain mathematical ideas and applications at a younger age than they might otherwise have done. For example, quite young children may make the calculator itself an object of study. The mathematics then becomes the theory that explains what functions the calculator performs and what rules it obeys. Older students may use graphing software to study the general shapes of families of functions and the effects of transformations on them, possibly achieving a richer understanding of functional relationships than has been possible in the past. Students should have sufficient experience of calculators and computers as mathematical tools to be able to make informed decisions about whether or not to use them in particular mathematical situations, and to use them efficiently when they wish to do so.

Increasingly, new developments in mathematics are described in 'lay terms' in magazines and television programs, and students should be encouraged to read and watch such material and link them to the work they are doing in school. Examples of relatively new mathematical ideas now becoming popularised in this way are fractal geometry and its relation, chaos theory.

While most people will never need to learn the more specialised forms of mathematics presently being created, school mathematics should reflect current developments in the field and present mathematics as a thriving, dynamic enterprise.

Students should experience the processes through which mathematics develops

The systematic and formal way in which mathematics is often presented conveys an image of mathematics which is at odds with the way it actually develops. Mathematical discoveries, conjectures, generalisations, counter-examples, refutations and proofs are all part of what it means to do mathematics. School mathematics should show the intuitive and creative nature of the process, and also the false starts and blind alleys, the erroneous conceptions and errors of reasoning which tend to be a part of mathematics.

Mathematical investigations can help students to develop mathematical concepts and can also provide them with experience of some of the processes through which mathematical ideas are generated and tested. They should be helped to gain insight into the nature of the subject and the motives which lead people to pursue it.

Studying the origins of particular mathematical ideas and techniques and the links between them can help students to integrate ideas which occur in apparently distinct areas of mathematics and also further their understanding of the nature of mathematics.

The universal nature of many mathematical ideas and the extensive use of symbolic notation to portray abstract ideas lead many people to the mistaken view that mathematics is culturally neutral and value-free.[27] Students need to develop an appreciation of the process of growth and change and the relationship between mathematics and society. In particular, they should understand that mathematics grows out of specific social and historical contexts. For example, the needs of commerce and war set the parameters for certain major developments in European mathematics, while the needs of social organisation adapted to local conditions set the scene for the development of Aboriginal kinship systems which are mathematical in nature. Cultural appreciation of mathematics in Australia should include an understanding of the richness of mathematical ideas which were part of Australian society before the arrival of Europeans.

The socio-historical study of mathematics should not, however, focus exclusively on the past. Students need to understand that new mathematics and new applications are constantly being developed. They should have the opportunity to apply their mathematics in settings which reflect the multicultural nature of Australia. They should also investigate a range of modern applications of mathematics to problems of concern to Australians, even if the technical demands of the mathematics are beyond them at the time.

Goals

As a result of learning mathematics in school all students should:

- realise that mathematics is relevant to them personally and to their community;
- gain pleasure from mathematics and appreciate its fascination and power;
- realise that mathematics is an activity requiring the observation, representation and application of patterns;
- acquire the mathematical knowledge, ways of thinking and confidence to use mathematics to:
  - conduct everyday affairs such as monetary exchanges, planning and organising events, and measuring;
  - make individual and collaborative decisions at the personal, civic and vocational levels;
  - engage in the mathematical study needed for further education and employment;
- develop skills in presenting and interpreting mathematical arguments;
- possess sufficient command of mathematical expressions, representations and technology to:
  - interpret information (e.g., from a court case, or media report) in which mathematics is used;
  - continue to learn mathematics independently and collaboratively;
  - communicate mathematically to a range of audiences;
- appreciate:
  - that mathematics is a dynamic field with its roots in many cultures;
  - its relationship to social and technological change

Enhancing mathematics learning

A great deal has been learned about how children and young adults learn mathematics and about the classroom conditions which are most supportive of their learning. While further research is needed to assist us to better understand the learning of mathematics, many of the innovations occurring in classrooms around Australia have considerable potential for improving mathematics learning. These experiences do suggest some general principles about learning which should inform our teaching and the learning environment we provide.

Learning principles

It is now widely accepted that learning is best thought of as an active and productive process on the part of the learner. There are several important implications of accepting this simple proposition.

Learners construct their own meanings from, and for, the ideas, objects and events which they experience

The meanings that people construct depend upon their existing understandings. We can only take from any situation the parts that either make sense to us or can be linked to some existing ideas we have. If a particular idea is not at all new to us, then no learning is necessary. Some learning can readily be accommodated within existing conceptual structures. Other learning requires a relatively simple extension or adjustment of ideas we already have. A considerable part of learning in mathematics, however, requires learners to change and expand their ways of thinking.

The source of mathematical knowledge is patterns and relationships created in the mind. Kamii,[28] for example, describes children's development of number concepts as the synthesising of order and hierarchical inclusion relationships:

> **The only way we can be sure of not overlooking any or counting some more than once is by putting them into some relationship of order ... The child does not, however, have to put the objects in a linear order; what is important is that he or she order them *mentally* in some way ...**

Ordering is insufficient, however, as is shown when young children point to the sixth bead in answer to a request to show six beads. What is also needed is that children:

> **... mentally include 'one' in 'two', 'two' in 'three', 'three' in 'four', and so on ... synthesising order and hierarchical inclusion.**

Number concepts are not physical properties and are not abstracted from collections of objects, but are a result of imposing logico-mathematical thinking on the collections:

> **... children become able to think about the objects as 'eight' only when they can impose their logico-mathematical knowledge, that is, self-created relationships, on the set.**

Thus, mathematic learning does not occur through the internalisation of external things; rather it involves the creation or building up of relationships in the mind of the individual.

Learning happens when existing conceptions are challenged

Learning is likely to occur if existing conceptions are challenged and the learner feels a need to accommodate the new information or ideas. The challenge may come from the physical environment, the social environment or from mathematics itself. An example of the first occurs when a student makes a prediction, experiments and finds it does not work. An example of the second occurs when a student is surprised to find that a peer disagrees with his or her interpretation of a mathematical situation. An example of the third occurs when a student believes that a particular rule will apply for all numbers and finds that this is not so. In each of these cases, learning occurs when the learner finds a way to accommodate the new perspective or information. As ideas are acquired they are used as a basis on which to acquire and make sense of other ideas.

Learning requires action and reflection on the part of the learner

Learning requires action on the part of the learner and reflection about those actions and experiences. The three situations described in the previous paragraph are unlikely to result in learning unless students think about and work on overcoming the disjunction between their current conceptions and the new information. Mathematics has its roots in activity and in reflection to guide and interpret that activity:

> **If you calculate without reflecting on what you have been doing, why you have been doing it, and what you have found, then your activity is likely to lose its sense of purpose and direction. If ... you try to speculate without getting involved in messy calculations, you are likely to lose touch with reality.**[29]

Reflection on experience is needed in order to link new knowledge to existing knowledge, leading to the expansion and refinement of ideas. The use of 'concept maps' in which students draw and discuss pictorial representations of the connections they have made between various mathematical concepts, examples, contexts, and so on, is one strategy which can help students reflect upon what they know and integrate new ideas with existing knowledge.

Learning involves taking risks

Learning involves taking risks. This means being willing to 'have a go', to guess, to try a new or different way of doing things. At times error can be a sign of progress, suggesting that the learner is prepared to work on new or difficult problems where increased error is likely, or to try improved ways of doing things which necessitate giving up old and safe but limited strategies. For example, a child who can consistently find six lots of 16 by adding has to take a risk in order to replace this strategy by a multiplicative one. Taking the risk may result in an increase in error in the short term but the long-term benefits are likely to be considerable.

One implication of taking risks is that mistakes are a necessary and acceptable part of learning. Errors provide a useful source of feedback, challenging us to adjust our existing conceptions before trying again. A second implication is that assessment strategies should not inhibit risk taking.

Implications for teaching

There is no definitive approach or style for the teaching of mathematics. The teaching of any particular mathematical concept will be influenced by the nature of the concept itself and by the abilities, attitudes and experiences of students. In general, however, teaching should be informed by a thorough understanding of how learning occurs and of the nature of mathematical activity.

Teachers assume a considerable responsibility for creating the best possible conditions for learning for all students. Recently, the Australian Association of Teachers of Mathematics in collaboration with the Commonwealth produced a *National Statement on Girls and Mathematics*[30] which addresses the issues of gender and mathematics. Other publications have focused upon issues of culture and the learning of mathematics in a second language.[31] Such materials offer guidance for improving the mathematics curriculum, both its content and pedagogy, in ways that make it more inclusive of the range of experiences and concerns of students in Australian schools.

Mathematics learning is likely to be enhanced by activities which build upon and respect students' experiences

While students in any particular classroom will have much in common, they also bring to the classroom a wide range of different experiences which should be valued and accommodated. For example, the skills associated with counting and quantifying are important for functioning in mainstream Australian life. In many families they are highly regarded skills and are in constant use. For the children of such families there is an inherent meaning and purpose in learning to count and operate on numbers. Some such children may enter school proficient in counting and simple computation and able to compare quantities. Counting and measuring are not, however, equally valued within all families, even when they are a part of daily life. Furthermore, symbolic ways of representing mathematical ideas are not identical in all cultures, even when the same ideas are being represented.[32]

Many students have difficulty with the various language forms used in mathematics classrooms. For some students for whom English is a second language, it may be beneficial to continue to learn mathematics in their first language until they have a better grasp of English. For all students, building on their experience implies building upon their English.

Building upon students' experiences is vital, but learning mathematics also requires going beyond the 'everyday'. If students are to gain access to the power and generality of mathematics it will not be sufficient for them to find intuitive solutions to individual 'real-world' problems. They will also need to 'read and write the symbol system precisely and to handle relationships simultaneously and as entities'.[33]

Mathematics learning is likely to be enhanced by activities which the learner regards as purposeful and interesting

If learning requires action on the part of the learner, then the opportunity to learn what is intended from an experience is likely to be enhanced if students see its purpose. How students respond to the task and what they learn from it may be quite different depending upon their conception of the task. For example, students need to know when the purpose of an activity is to provide drill to increase the accuracy and efficiency with which they can perform an already learned procedure, and when the activity is one which may demand persistence, thoughtfulness and reflection.

Young children usually show an uninhibited enthusiasm and curiosity; school is enjoyable and so is mathematics. The challenge through the years of schooling is to present mathematics in a way which continues to be interesting, challenging and rewarding. Students should have the opportunity to use mathematics in decision making and problem solving about situations that are of interest in their own right and not simply because they demonstrate some mathematics particularly well. They should also have the opportunity, throughout *all* years of schooling, to investigate mathematical situations of their own generation in order to experience the pleasure of investigating their own problems and the satisfaction of reaching their own conclusions.

Mathematics learning is likely to be enhanced by feedback

If learning happens when existing conceptions are challenged then feedback is essential for learning. Feedback may take many different forms. As was suggested earlier, it may come from the physical, social or the mathematical environment. Clearly, the role of teachers is critical in this process.

Often appropriate feedback can help students to recognise inconsistencies in their own thinking. For example, a student may continue a sequence made by repeatedly adding 0.2 thus:

$$0.2,\ \ 0.4,\ \ 0.6,\ \ 0.8,\ \ 0.10,\ \ 0.12, \ldots$$

Completing the sequence with a calculator produces the following:

$$0.2,\ \ 0.4,\ \ 0.6,\ \ 0.8,\ \ 1.0,\ \ 1.2, \ldots$$

This inconsistency then has to be dealt with. Little is learned if students simply mark their sequence wrong and turn to the next one. In order to use these inconsistencies to further their own understanding, students have to believe that mathematics makes sense, that it is not just a series of arbitrary rules, and that they can work it out.

Discussion with others is one means of finding out that others do not share our point of view; we constantly adjust our understanding and interpretation of phenomena through our interactions with other people. Through discussion with peers, teachers and others, students may adjust their conceptions by gaining new information or as a result of scepticism on the part of the others. Often discussion will involve students in explaining ideas to others and, in so doing, clarifying them for themselves.

Mathematics learning is likely to be enhanced by using and developing appropriate language

Students should learn to use language as a tool for reflecting on their mathematical experiences and hence for their own mathematical learning. Explaining to oneself, 'putting it into words', can be a powerful means of working through and clarifying ideas. Mathematical concepts are not developed in the absence of mathematical language. Students are likely to develop mathematical ideas more readily when they have clear ways of labelling and talking about their experiences.

Explicit attention needs to be paid to the development of mathematical expression. Teachers might make a point of regularly using particular terms or language patterns so that students hear them in situations which make their meaning clear. Students can be encouraged to practise their own use of the language by describing experiences orally and in writing.

There should be a gradual reduction in ambiguity through the use of conventional language and improved clarity of expression. This should build on students' everyday language but, at times, quite explicit attention may need to be paid to the meanings students have developed for words. For example, a word may have an everyday meaning which is different from its mathematical meaning (e.g. reflection, irrational). Alternatively, an over- or under-generalisation of the meaning of a word may have been made (e.g. not realising that squares are special kinds of rectangles).

The careful development of their capacity to interpret and use mathematical expression is of particular relevance to children from non-English speaking backgrounds (NESB). It appears, however, that the majority of all children benefit considerably from the approaches to language development in mathematics which have resulted from work with NESB children. This is particularly so for students who have previously encountered little formal English.

Students also need to develop the skills of recording their mathematics. The first forms of recording are likely to be in everyday language or in pictures or diagrams. Gradually these representations may be shortened, leading to the need to use symbols. At each stage of schooling, the same gradual development from the familiar (more 'concrete') to the more abstract ideas and representations is needed.

Mathematics learning is likely to be enhanced by challenge within a supportive framework

A challenging environment means that risk taking is encouraged and that all students are extended appropriately. Clearly, repeated failure is not a good recipe for developing confidence or competence. The provision of achievable but challenging tasks and the experience of success are crucial in building positive attitudes towards mathematics.

If learners encounter continued success on personally easy or rote tasks, however, they may become less and less able to take the risks needed for higher level learning. Even though they may be 'getting the answers right', they lose faith in their capacity to deal with unfamiliar or challenging tasks.[34] There is evidence that such students are more likely to be girls than boys. It appears that students' successes and failures and their behaviour tend to be interpreted differently depending upon whether they are girls or boys.[35] Thus, a girl's successes are somewhat more likely to be interpreted as due to conscientiousness and hard work, while a boy's successes are more likely to to be interpreted as 'natural' ability. Teachers, parents and the students themselves may all do this to some extent. Many girls, particularly mathematically very able girls, accept these interpretations themselves. They often believe that their successes are due to hard work alone, and that they are not as able as other equally achieving students. Consequently, their belief in their capacity to keep succeeding is undermined.

Students should experience the reward of arriving at solutions through their own initiative and persistence and not simply through imitation. The challenge for teachers is to help students learn to seek imaginative solutions to problems in constructive ways, rather than to avoid all stress and struggle.

Assessment

Assessment is an integral part of the learning process. Indeed, the major purpose of assessment is the improvement of learning. Assessment provides feedback about students' mathematical development to students and their teachers. This feedback should inform the future action of both learners and teachers. Assessment can also be used to report students' progress to parents, prospective employers, and other educational agencies.

Assessment should reflect all of the goals of the school mathematics curriculum

What students learn and how they learn will be influenced by what they think teachers and examiners value. Their view of what really counts in learning and doing mathematics will relate quite closely to what is assessed. If we wish students to develop a range of knowledge, skills, ways of thinking and habits of thought in mathematics, then we have to ensure that we are assessing how well they are doing so.

Teachers' practices will also be influenced by the results of student assessments. Student assessment provides feedback to teachers about the effectiveness of their past actions and suggests directions for future action. If some areas of the curriculum are assessed to the exclusion of other areas, teaching will tend to emphasise those, whether consciously or not.

For these reasons, assessment should address all the goals of the curriculum. One implication of this is that we need to develop strategies for the assessment of some of the newer content of school mathematics. For example, we need to assess students' capacity to judge the reasonableness of results and to choose appropriate levels of accuracy, data collection and analysis techniques, mathematical investigation and modelling. We also need to assess the development of attitudes and appreciations. It is essential that we improve our assessment practices so that they more adequately reflect all of the purposes of school mathematics.

Assessment should be demonstrably fair, valid and reliable

Assessment is used to make decisions about students which will affect their future learning and their educational and occupational options. Clearly, it should be fair, valid and reliable.

In the past, mathematics has been assessed mostly by paper-and-pencil timed tests. This form of testing has remained dominant in mathematics, partly because it was perceived to be more objective and hence more fair than other forms of assessment

The fairness of testing only those aspects of mathematics which can readily be assessed in traditional test questions can easily be challenged. The strengths of some students will be favoured by short-answer questions which rely on quickness of mind and speed, the strengths of others by extended-response questions which rely on reflection and persistence. Students may not achieve equally well on all aspects of the mathematics curriculum. Some may do particularly well with reasoning tasks, while others have exceptionally good memories. Some will develop better spatial skills, others a better understanding of number. Consequently, inferring achievement in mathematics generally from a non-representative sampling of the curriculum outcomes or through a narrow sampling of methods of assessment may be unfair to many students.

Paper-and-pencil tests, with or without a time limit, are likely to continue to be one useful and efficient means of gaining feedback about student learning. It is clear, however, that conventional forms of such tests cannot address all areas of the mathematics curriculum. Additional modes of assessment must be developed. We will need to increase our repertoire of types of questions which can be asked in traditional test settings and develop strategies for judging the responses to these questions. In addition, other methods must be developed which are fair, valid and reliable and which are seen to be so.

While more developmental work is needed on useful, practical and fair assessment strategies, a range of practices which can be used to gather information have been identified.[36] They include: teacher observation and questioning; structured interviews with students; paper-and-pencil tests; oral tests; practical skills tests; work- or project-based assessment; collected samples of students' independent work; individual homework assignments; group reports; anecdotal records; self-assessment; and peer assessment.

Supporting mathematics learning

The view of mathematics and the approaches to teaching and learning suggested in this *Statement* have implication for how mathematics learning is supported. For example, they suggest a classroom learning environment which encourages practical activity, the appropriate use of technology, and discussion. Mathematics can no longer be regarded as a 'chalk and talk' subject from the perspective of the teacher or as a 'textbook, pencil and paper' subject from the perspective of the students. The *Statement* also has implications for the professional development of teachers and for the nature of parent and community involvement in school mathematics.

The classroom environment

Classes should take place in settings which provide a rich mathematical environment. Ideally, they should enable the flexible use of furniture so that movement and discussion can proceed without excessive noise disturbing others. Resources should include a wide range of locally collected or produced materials and commercial products. Resource materials should reflect the diversity of Australian society and help students understand the range of uses of mathematics, even if the mathematics is technically out of their present reach, and should encourage students' participation in mathematics. Student work should be displayed regularly and school mathematics should be seen to be a thriving and dynamic enterprise, like mathematics itself.

Many Australian students do not undertake their schooling in 'regular' classrooms, because of geographic isolation, physical disability or possibly illness. In such cases, providing appropriate resources can be difficult. Part of the solution may lie with local materials, part with the use of computing and communications technology.

Creative solutions should be sought to ensure that each student experiences a mathematically rich environment.

Use of technology

In 1987 all Australian education systems endorsed a national calculator policy,[37] which began as follows:

> **It is recommended that, as far as resources allow, teachers should:**
>
> 1. **ensure that all students use calculators at all year levels (K-12);**
> 2. **ensure that the calculator is used both as an instructional aid and as a computational tool in the learning process;**
> 3. **be actively involved in the curriculum change in content and methods arising from calculator use;**
> 4. **take full advantage of the potential of calculators for mathematics within the total curriculum;**
> 5. **initiate discussion locally regarding the role of calculators in the school and society.**

The present *Statement* provides further endorsement of those recommendations. Since that time there have been considerable advances in the computing software suitable for use in school mathematics. There is a range of ways in which computers may be used in mathematics classrooms including 'number crunching', data analysis, as a simulation device, graphics, symbol manipulation and spreadsheets.[38] Each of these uses has implications for what is most usefully taught in mathematics and how it is taught. If the use of computers is to be regarded as a normal part of doing mathematics then students will need access to computers in the rooms where mathematics is normally learned. A range of other technologies such as television and video may also be needed to enrich students' mathematical experiences.

Text materials

Schools need a variety of print materials. The dynamic nature of the mathematics curriculum and the need to cater for the interests of all students mean that no single text is likely to cater for all students. Special care needs to be taken that the material selected does not conflict with the values of some groups in Australian society and that gender and racial stereotyping are avoided. The language used should be appropriate to the majority of students while encouraging development of mathematical expression. Some students will need additional support in dealing with mathematical texts. Resource teachers, especially English as a second language teachers, must be involved in the teaching of mathematics.

The teaching profession

Teachers play a key role in supporting the learning of mathematics. Over recent years many changes have occurred in the content and methodologies for teaching mathematics and in community expectations about mathematical outcomes. These changes are likely to continue. Teachers, like other workers, will need to be engaged in professional development throughout their careers if they are to keep abreast of these developments. The *Discipline Review of Teacher Education in Mathematics and Science*[39] has made specific recommendations on many aspects of the preparation and continuing education of primary and secondary teachers of mathematics, as well as commented on the present situation nationwide. Consequently, this whole issue is not addressed in this *Statement*.

Parent and community involvement

Mathematics learning is enhanced when students feel that their parents are concerned about the learning program in which they are involved. Parents of younger students may often know and understand the learning program and be able to encourage, reassure and assist the learner in a positive way. For older students assistance may not always be possible, but encouragement and support remain vital. In particular, students need to feel that their parents value the things they are learning in school.

Students are likely to respond more positively to the experiences they have in school if they feel that those experiences relate to the lives of their communities. The involvement of the school in the life of the community and the community in the life of the school is vital in this regard. The wider community is part of the broader environment in which mathematics learning occurs. Often the views children and young adults have of mathematics and of themselves as learners of mathematics are formed outside the classroom. Parents and the wider community play an important role in ensuring that mathematics is regarded as important and accessible by all young people.

The Scope

of the mathematics curriculum

Organising the mathematics curriculum

The organisational structure presented in Part II is provided by a framework of eight strands: **Attitudes and appreciations**, **Mathematical inquiry**, **Choosing and using mathematics**, **Space**, **Number**, **Measurement**, **Chance and data**, and **Algebra**. Within each strand, mathematics for primary and secondary years is described using four broad bands of schooling: A, B, C and D.

A mathematics curriculum may be structured around themes, problems, contexts or areas of application, or fields within mathematics. There are many possibilities. The organisation of this *Statement* is not intended to promote any one particular way of organising curriculum or the learning experiences provided in classrooms. However, it does emphasise the major kinds and sources of mathematical activity and makes explicit the links between the mathematics learned at various stages of schooling.

The strands

Learning mathematics involves learning both its products and its processes. The products (*body of knowledge*) are the facts, concepts and generalisations and the standard models and procedures of mathematics. The processes (*ways of knowing*) are the mathematical thinking skills which enable the products to be developed, applied and communicated. In some strands the products of mathematics are emphasised, while in others the processes are emphasised. In order to understand the nature of the mathematics and what distinguishes it from other human activity, students need to appreciate and experience the interplay between its products and processes. Therefore, while there may be advantages at times in emphasising one or the other, in practice they will usually be interwoven and their relationship to each other emphasised.

Partly for historical reasons and partly because of the complexities of course design, mathematical ideas which are conceptually linked are often presented and learned quite separately. Effective problem solving, however, requires that students be able to integrate mathematical ideas and draw on the most appropriate analytic devices for the task at hand. It is therefore important that students are helped to see the links between ideas and procedures that occur in different areas of mathematics.

There are significant overlaps and inter-connections between the strands, which should be drawn out in practice. For example: measurement concepts should be developed together with those of space and number; algebra provides tools which may be needed in all other areas of the mathematics; and concepts such as that of *function* are fundamental in mathematics and will appear in different forms in all strands.

While the document suggests a development of mathematical ideas as students progress through school, these are described in very broad terms only. Clearly, curriculum planning must also take into account the sequencing of mathematical ideas both within and between strands.

All strands reflect and anticipate changes needed in mathematics teaching in this decade. The eight strands, which are further broken down into subheadings, are as follows.

Attitudes and appreciations

Attitudes deals with the development of positive attitudes towards mathematics and students' involvement in it.

Appreciations deals with the development of an appreciation of the nature, power and scope of mathematical activity.

Mathematical inquiry

Mathematical expression deals with interpreting and conveying mathematical ideas.

Order and arrangement deals with observing and generalising patterns and relations.

Justification deals with explaining and justifying conclusions leading to the notion of proof.

Problem-solving strategies deals with a range of strategies for problem posing and solving.

Choosing and using mathematics

Applying mathematics deals with choosing and using standard mathematical techniques in situations in which mathematics may be useful.

Mathematical modelling deals with the more general processes by which 'real world' phenomena are represented in order that mathematics may be applied to them.

Space

Shape and structure deals with the properties of two- and three-dimensional objects and the relationship between shape, structure and function.

Transformation and symmetry deals with the mathematical equivalent of changes of position, orientation, size and shape, and with symmetries in shapes and arrangements.

Location and arrangement deals with the representation of position and arrangement, including the use of coordinates.

Number

Number and numeration deals with concepts of number and the ways we write them.

Computation and estimation deals with the operations of addition, subtraction, multiplication and division and their application.

Measurement

Measurement and estimation deals with the comparison of qualities of objects, the use of units of measurement, and measuring and estimating skills.

Indirect measurement deals with the use of rates, measurement formulae, scale, angular measure and trigonometric ratios for indirect measurement.

Approximation, change and the calculus deals with rates of change, and infinite and limiting processes.

Chance and data

Chance deals with the concepts of randomness and the use of probability as a measure of how likely it is that particular events will occur.

Data handling deals with collecting, organising, summarising and representing data for ease of interpretation and communication.

Statistical inference deals with drawing conclusions and making predictions based on both data and principles of chance.

Algebra

Expressing generalisations deals with algebraic expressions as generalised statements.

Functions deals with the general statement of relationships between quantities and techniques for graphing.

Equations deals with setting up and solving equations and inequalities.

The bands

Students come to school with a diverse range of attitudes and experiences. They may also learn at different rates and in different ways. Therefore, it is not possible to equate the bands A, B, C and D with specific year-levels of schooling. It is anticipated, however, that the typical child will need experiences mainly from band A in the lower primary years, from band B in the upper primary years, and from band C in the lower secondary years. Some students may need experiences from band A throughout their primary years and from band B in the lower secondary years; others may gain from band C experiences in the upper primary years and some band D experiences in the lower secondary years.

Bands A to C contain all the experiences considered necessary for the typical citizen. Our goal should be for all students to gain access to the mathematics within those strands, albeit in different ways and at different paces. Increases in school retention rates make this goal more achievable than it once seemed. While band D extends mathematical knowledge beyond that needed for the typical citizen, it also recognises the wide range of contexts in which students may wish to function in the future.

The *Statement* should be interpreted in such a way that the interests and capabilities of every student are fully stimulated.

The scope of the curriculum

Chapters 5 to 12 of this *Statement* describe the scope of the curriculum in terms of the mathematical understandings, skills, knowledge and processes which should typically be made available to students.

The discussion of each strand begins with a general introduction which describes the purpose and importance of work within that strand and the major mathematical ideas to be developed during years 1–12. This is followed by a more detailed discussion of the strand under each of the four bands.

Within each band in each strand, a series of *scope statements* provides the framework around which States or schools may build their mathematics curriculum. These scope statements are listed A1, A2, A3, and so on, for band A, through to D1, D2, . . . , for band D. The letters indicate the band and the numbers provide a simple labelling of the scope statements. *There is no implication that scope statements within a band should follow the sequence presented.*

The lists of activities which follow each scope statement are intended to provide clarification and support; they should not be regarded as either comprehensive or restrictive.

Experiences have not been described for the pre-compulsory years because of the structural differences in provisions between States and Territories. These years, however, are vital in children's mathematical development. People involved in pre-compulsory education are encouraged to use the *Statement*, particularly band A, as an indication of broad directions and emphases for the pre-compulsory years.

The post-compulsory years

The post-compulsory years provide for students an intermediate step between the more structured educational experiences of the compulsory schooling years and the broad variety of learning experiences which are present in adult life. During these years students should have access to courses in which all strands are represented and the content of these courses should allow for different treatments of the strands appropriate to the needs of students.

The mathematics of the post-compulsory years must build upon students' experiences in mathematics during the compulsory years. This means that included within the range of courses available should be those which draw on mathematics from band C, and possibly also band B. Students in these years can be expected to have new priorities and require new experiences. Therefore, courses designed for students in the post-compulsory years may draw on mathematical experiences from bands B and C, but should do so in a way which reflects the added maturity of the students, their proximity to adult life and their aspirations.

Other courses in the post-compulsory years should draw on experiences from band D. These experiences are appropriate for students who have successfully undertaken those in band C. They are at a higher level of sophistication, more abstract, more analytical, and also more demanding of independent mathematical work.

Students for whom experiences drawn from band D are appropriate will vary in their aspirations, their intrinsic interest in mathematics, and their present capabilities. Mathematics courses which draw mainly on experiences from band D should reflect the range of student needs, interests and aspirations.

Some such students will wish to continue the study of mathematics in further education. They may study mathematics in higher education, for its own sake, or for computing, physical science or engineering. Others may study mathematics for the social sciences, such as economics, geography, sociology or psychology, or for professional work in health, veterinary or education fields. Still others may undertake further vocational studies which will rely to an extent upon mathematics.

Some students may anticipate no further formal study of mathematics; they may wish to be lawyers, journalists, fashion designers, chefs or landscape gardeners. These students should be also be encouraged to study mathematics which they find challenging and personally rewarding in order to enhance their potential to fully participate in and contribute to society.

The scope statements provided for band D reflect significant mathematical experiences within each strand, but are described in more general terms than those for bands A to C. The activities provided with each scope statement indicate the range of ways in which the scope statement might be realised. Certainly, not all activities would be appropriate or desirable for all students and it is not intended that all the mathematics in a particular strand would be studied by all students. Neither is it intended that the mathematical experiences cited should restrict the range of experiences made available within any strand.

Attitudes and appreciations

Purpose and scope

An important aim of mathematics education is to develop in students positive attitudes towards mathematics and their own involvement in it, and an appreciation of the nature of mathematical activity. The notion of having a positive attitude towards mathematics encompasses both liking mathematics and feeling good about one's own capacity to deal with situations in which mathematics is involved. Both are desirable but the latter is essential! Students also need to learn *about* the nature of mathematics and the processes through which it is produced and applied.

Students will vary in their interests: some will enjoy mathematics more than others. A particular student may enjoy some aspects of mathematics but not others. It is unrealistic to expect all students to find the same experiences pleasurable. Nonetheless, there is considerable evidence to suggest that children come to school enthusiastic and eager to learn mathematics and that a great many leave school with quite negative attitudes. Some dislike the subject, others feel inadequate about it, still others feel that it is irrelevant in their lives. This is an unacceptable situation for reasons canvassed throughout Part I. The development of attitudes and appreciations is an important aspect of learning mathematics and must be addressed quite explicitly in the teaching of mathematics.

Thus, students should have the opportunity to appreciate mathematical products and processes as well as their application to the social and physical environment, and to experience the pleasure and the fascination of investigating their own problems and the satisfaction of reaching their own conclusions.

The scope statements for this strand have not been differentiated by band, although appreciations can be expected to increase in sophistication as students progress through school. They have, however, been arranged under two subheadings, *Attitudes* and *Appreciations*.

Attitudes deals with the development of positive attitudes towards mathematics and students' involvement in it.

Appreciations deals with the development of an appreciation of the nature, power and scope of mathematical activity.

The activities listed with each scope statement all appear in one or more of the other strands. They represent a small selection from those available. The purpose is to demonstrate that the development of appropriate attitudes and appreciations should be an explicit consideration in the choice of activities, but that these activities should occur in all strands and all bands.

ATTITUDES

Throughout all bands, experiences should be provided to encourage in students:

1 a positive response to the use of mathematics as a tool in practical situations

Possible activities:

- Apply mathematics to practical situations which are not necessarily mathematical (e.g. in making a book about oneself, measure and collect, organise and interpret data, design pages and decide how many pages, how big, how to fold). **Choosing and using mathematics**, A1
- Discuss and make judgements about arguments and claims in the media which present statistical information (e.g. a claim that 40% of the community think that the school leaving age should be raised, based on a telephone 'ring-in' poll). **Chance and data**, B6
- Convert currencies from one to another, and research the movement of currencies over a short period, deciding (retrospectively) the day which would have provided the best yield for a given exchange. **Number**, C7
- Solve design problems (e.g. use tessellation in the plane, the relationship between shape and function …) using drawing instruments and CAD packages. **Space**, D3

2 the confidence to apply mathematics and to gain knowledge about mathematics they need

Possible activities:

- Frame questions which ask for clarification or show areas of misunderstanding (e.g. I don't think you can take 8 away from 6 because when I punched 6 – 8 on the calculator it gave me a 2 with a 'take away' sign in front of it; what does that mean?). **Mathematical inquiry**, A2
- Research and report on some of the different ways numbers are used as labels (e.g. postcodes and ISBNs). **Number**, B1
- Read in order to find out about some mathematics needed in problem solving (e.g. read a text which explains how to operate with indices needed for a problem about a bouncing ball). **Mathematical inquiry**, CD1

3 a willingness and ability to work cooperatively with others and to value the contributions of others

Possible activities:

- Make informal maps and refine through use with other children. **Space**, A9
- Use simple procedural rules to enable groups to work effectively on the solution of problems. **Mathematical inquiry**, B9
- Work collaboratively and pool information to make order of magnitude estimates (e.g. Fermi problems: how many cups of coffee are drunk in Australia each year?). **Choosing and using mathematics**, CD5

4 a willingness to persist when solving problems and to try different methods of attack

Possible activities:

- Discuss the 'errors' and 'wrong' ideas which have been helpful in reaching a final solution. **Mathematical inquiry**, A6
- Discuss the results of investigations, comparing methods used and false trails (e.g. patterns that 'didn't continue'). **Mathematical inquiry**, B8

- Discuss different strategies applied to the same problem, comparing successful strategies and noting unsuccessful strategies. **Mathematical inquiry**, CD8

5 an awareness that creativity and initiative are encouraged and valued within mathematics education

Possible activities:

- Judge the appropriateness of a solution in a given real situation (e.g. I think we asked the wrong question: why would we want two sets of the same 10 colours?). **Choosing and using mathematics**, A3
- Generalise a problem solution in order to make predictions about related problems (e.g. having found the number of diagonals for a pentagon, hexagon, septagon, predict for an octagon). **Mathematical inquiry**, B3
- Given a starting situation (e.g. a spiral on squared paper) brainstorm questions about it and choose some questions to investigate. **Mathematical inquiry**, CD9

6 an interest in and enjoyment of the pursuit of mathematical knowledge

Possible activities:

- Carry out open-ended explorations in number (e.g. find patterns on the 'one hundred chart'). **Mathematical inquiry**, A8
- Compare natural shapes (e.g. leaves of plants) from different environments, considering the adaptations of shape to function. **Space**, B2
- Investigate number patterns in nature, art and architecture, considering their aesthetic and utilitarian purposes, and their history. **Number**, C1
- Carry out mathematical investigations which involve cycles of observation, conjecture and justification on topics of interest. **Mathematical inquiry**, CD10

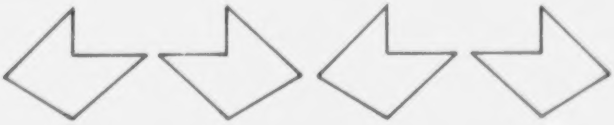
APPRECIATIONS

Throughout all bands, experiences should be provided to enable students to appreciate:

7 that mathematics involves observing, generalising and representing patterns

Possible activities:

- Recognise, copy, continue and devise simple repeating patterns of numbers and represent these patterns in spatial designs. **Mathematical inquiry**, A4; **Number**, A3
- Describe simple spatial sequences and predict elements (e.g. sequence of shapes). **Algebra**, AB1



- Use a calculator to explore number patterns given a starting place, e.g. explore and explain the following pattern:

$$6^2 - 5^2 =$$
$$56^2 - 45^2 =$$
$$556^2 - 445^2 =$$

Mathematical inquiry, CD3

- Graph data and determine an appropriate mathematical model (e.g. repair costs related to vehicle age, cattle weight related to age) and use the model to make predictions. **Choosing and using**, C6

8 that the economy and power of mathematical notation and terminology help in developing mathematical ideas

Possible activities:

- Devise shorthand ways of representing the results of combining, separating, grouping and sharing collections. **Mathematical inquiry**, A1
- Investigate verbal expressions which may be ambiguous (e.g. three oranges and apples) and restate to remove the ambiguity and relate to the development of precise language in algebra (e.g. distinguish between $3(a + b)$ and $3a + b$). **Algebra**, C1
- Work collaboratively to explain the meaning of mathematical terms and expressions (e.g. hypotenuse, if and only if), redrafting to remove ambiguity. **Mathematical inquiry**, CD2
- Construct and interpret simple route matrices to describe given networks and vice versa (e.g. this matrix represents the traffic flow in a city street system with A, B, C and D as corners; which are the one-way streets?). **Algebra**, D1

9 that conventions, rules about initial assumptions, precision and accuracy enable information to be shared unambiguously

Possible activities:

- Engage in activities in which there is a need to communicate measurements (e.g. shop for the tape needed for Easter bonnets) in a standard way. **Measurement**, A3
- Choose and execute appropriate checks on accuracy (e.g. by repeating the calculation in a different order, carrying out rough approximation, and using patterns of final digits). **Number**, B12
- Investigate calculator key sequences which are ambiguous because they give different results on two types of calculators, and rewrite the expressions to remove the ambiguity. **Algebra**, C1
- Evaluate reports and media articles where information has been collected, summarised and interpreted (e.g. phone-in polls, popular scientific magazines, news items on sports, politics, economy). **Chance and data**, D4

10 that a mathematical model is a simplified image of some aspect of the social or physical environment

Possible activities:

- Make judgements about whether the conclusions reached make sense in the situation (e.g. When I added the number of us who play tennis to the number who play hockey, I got more than there are in our class: this doesn't make sense, so I need to check what I did). **Choosing and using mathematics**, B4
- Discuss models which have come to assume the status of 'real-world' phenomena, such as IQ, inflation, tertiary entrance scores, strike rate, batting average, considering what reality these models are intended to represent. **Choosing and using mathematics**, CD4
- Examine properties of probability models (e.g. binomial, Poisson, normal, ...) and consider the assumptions and hence the limitations of each. **Chance and data**, D2

11 that rigorous justification of intuitive insights is important

Possible activities:

- Test conjectures (e.g. we made different shaped rectangles each from 12 tiles and found that they didn't all have the same length around the outside). **Mathematical inquiry**, A5
- Test conjectures by systematically checking all cases (e.g. check all the rectangles that can be made with 36 squares to demonstrate that the square has the smallest perimeter). **Mathematical inquiry**, B4
- Draft and redraft a proof of a proposition until convinced it will withstand the scrutiny of peers and submit to the scrutiny of peers. **Mathematical inquiry**, CD6

12 that mathematics has its origins in many cultures and is developed by people in response to human needs, both utilitarian and aesthetic

Possible activities:

- Play counting games and sing counting songs from various cultures. **Number**, A2
- Investigate various ways people have measured and do measure time, and different calendars, past and present. **Measurement**, B4
- Investigate the role of statistics in shaping and describing aspects of public life (e.g. forecasting economic trends, describing and influencing consumer tastes, developing public perception and public policy on controversial issues). **Chance and data**, C8
- Research the origins of different geographic map projections, the mathematics underlying them and the distortions inherent in them. **Space**, D1

Mathematical inquiry

Purpose and scope

This strand addresses communication skills, ways of thinking and habits of thought which are explicitly, although not exclusively, mathematical. *The mathematical processes described within this strand cannot be developed in isolation from the work of other strands. They should pervade the whole curriculum,* and students should come to realise that it is through these processes that mathematics is generated.

The strand is described under four subheadings: *Mathematical expression; Order and arrangement, Justification;* and *Problem-solving strategies.*

Mathematical expression

Mathematics is one of the many ways of communicating information. It is better at communicating some sorts of information than others and makes some things appear more systematic and comprehensible. While it has its own jargon, it is not different in kind from other uses of the ambient language (that is, English, Italian, Vietnamese, ...). In doing mathematics, however, we use a range of special symbols, technical vocabulary and diagrammatic representations. We also give different meanings or special emphases to otherwise common words and combine words in particular ways, emphasising certain grammatical structures over others.

Unlike everyday language, the formal language of mathematics is concise, with little redundancy or ambiguity. These features, which are part of the power of mathematics, also make it difficult to read and write. The task for the learner is to relate the reduced form in which mathematics is presented to everyday language. Often it is assumed that a representation 'speaks for itself' when this is not the case. Consider, for example, the following exercise: complete $6 + [\;\;] = 11$. To make sense of this, students need to move from this symbolic proposition to the linguistic proposition — if six added to a number is equal to eleven, then what is the number? Many children who can 'fill in the box' correctly will, nonetheless, tell you that the *answer* is eleven. This may be because they don't know what the question is or because they have come to think of the symbol '=' as meaning 'what is the answer?' or 'makes'.

Diagrammatic and pictorial representations suffer from the same difficulties: they do not 'speak for themselves'. Since the conventions underlying pictorial representations are related to culture, it should not be assumed that Western conventions are universal or that all children will immediately recognise them. The conventions of drawing and the language of diagrams need to be learned.

The redundancy of everyday language means that it is not necessary to fully understand the meaning of every word or phrase to understand the overall message. The lack of such redundancy in mathematical text means that we need to know the meaning of every word or phrase in order to fully understand the message. The text will be very 'dense' in meaning. Encouraging students to read full sentences, at least initially, may reduce the density of the information and increase the predictability of the text (e.g. $6 \times 4 = 24$ is read as 'six multiplied by four is equal to twenty four'). It may also help them to develop their command of mathematical forms of expression. Oral and written expression in mathematics should be grammatically correct and to the same standard as would be expected in any other field.

If students are not to be restricted in their future mathematical opportunities they need to be able to read and to articulate mathematics.

Order and arrangement

A characteristic of humans is that we impose patterns upon things we see and do. Finding regularity in our physical and social environments is aesthetically appealing and also enables us to understand and make predictions about them.

Underlying the use of mathematics to understand our world is the recognition that different situations may share the same underlying form or pattern. For example, the 'dots' and 'dashes' of the usual visual representation of Morse code may be understood in a variety of different sound forms. We can hear the idiosyncratic features of the different sound versions of the code and at the same time recognise the underlying sameness of pattern. Observing the pattern itself (that is, disembedding the pattern from its context) is part of the essence of doing mathematics. Mathematics also provides us with very effective means for representing these patterns so that they may be investigated independently of their original context. The advantage of investigating patterns disembedded from their contexts is that conclusions reached about the pattern can be applied to all the other contexts in which the pattern is observed.

As they proceed through school, students should develop their abilities to observe, investigate and represent patterns of increasing complexity.

Justification

A justification is an attempt to convince people of the truth of a statement. Conjecture, explanation and justification (including proof) are presented in order to communicate a position and to invite criticism, modification and judgement. Cycles of conjecture and proof, accompanied by the scrutiny and judgement of peers, play an important role in the growth of ideas in mathematics.

Students often will 'see' generalised patterns on the basis of very little information and see no need to go further. As they mature, students should be helped to see the need to confirm their generalisations. They should be asked 'Does it always work?' and encouraged to test their generalisations, perhaps with more numbers or with different kinds of numbers. Some writers have suggested that students be exhorted to 'Convince yourself, convince a friend, convince an enemy!'.

Students should also know that a mathematical generalisation is something which is true, not *mostly*, but *always* and that one exception invalidates the generalisation. There should be an increasing readiness to seek and to offer clarification, to test the limits of their generalisations and to demand rigorous justification of their intuitive insights. Gradually, students' understandings of what constitutes a good explanation and a good justification should become more sophisticated and they should recognise the difference between a special case, an explanation and a general argument. They should eventually learn to form general explanatory statements of why 'it must be so'. Finally, students should compare different notions of proof, for example in law and in mathematics. Within mathematics, they should distinguish statistical arguments from proof by deductive reasoning or by exhaustive test. Some students, particularly in band D, should also use a range of proof techniques.

Problem-solving strategies

Students' problem-solving capacity will be be supported if they are able to select from amongst a repertoire of heuristic strategies, for example:

- guessing, checking and improving;
- looking for patterns;
- making a model or drawing a picture;
- making an organised list or table;
- restating the problem;
- separating out irrelevant information;
- identifying and attempting subtasks;
- solving a simpler version of the problem;
- eliminating possibilities.

Any of these may be helpful for a particular problem.

A repertoire of strategies is likely to be developed by solving a wide range of problems and also by seeing that a particular problem may be solved using a number of different strategies. It is likely to be insufficient simply to provide a great many problems, because there is no way of ensuring that students use a range of strategies over time, particularly since most students will have a number of different teachers. Teachers will need to monitor student experiences and encourage the use of a range of strategies. As students' experience grows they should become more efficient at choosing helpful strategies.

Whenever students solve a problem they should be encouraged to extend the problem in one of two ways. The first way is to focus on the strategy (e.g. Where else might guess and check be useful?) and the second way is to pose a related problem (e.g. Would it happen with triangles?). Teaching students to think about their thinking is the key.

Problem solving is also assisted by the development of personal management qualities which assist in individual and collaborative problem solving. Students should engage in extended mathematical activities which encourage problem posing, divergent thinking, reflection and persistence. They should be expected to pursue alternative strategies, and to pose and attempt to answer their own mathematical questions.

The methods that good problem solvers use to help themselves get started and keep going are likely to be idiosyncratic, and students should think about and talk to each other about what works for them, thus raising their awareness of the range of techniques available.

MATHEMATICAL INQUIRY in BAND A

Overview

From an early age children sort, classify and organise information and classroom experiences. They should build upon and extend these processes. Children will classify according to many different criteria, which may be mathematical (e.g. shape, size, direction, orientation) or not (e.g. like/dislike, red/green, toy/non-toy). The essence of work in this band is the observation and creation of patterns and relationships and their expression, initially in everyday language and gradually in more mathematical forms.

The language patterns necessary for linking a range of experiences to a single mathematical concept and its symbolic representation also need to be developed. Many verbal statements, such as:

I took three pens and then two more pens and then I had five pens

should be expected before children can readily say:

Whenever I take three and then two more, I have five

and this should precede the writing of symbolic statements, such as:

$3 + 2 = 5$

Similarly, the oral language of fractions (e.g. Give me half) may be used and understood considerably earlier than the symbolic language of fractions (e.g. $^1/_2$). Children should write and draw about what they have done and found, gradually arriving at the conventional symbols.

Learning to read mathematics should be regarded as part of learning to read generally. Explicit attention may need to be paid, however, to the various alternative everyday language expressions which can be used for the one symbolic expression (e.g. 12 – 5 can be expressed as 'twelve take away five', 'twelve subtract five' or 'the difference between 12 and 5').

Young children have considerable curiosity and their experience of mathematics should reinforce this. They bring to the classroom already acquired problem-solving strategies which they have developed through games, puzzles and explorations of their environment. Exploration of both new and familiar mathematical ideas and concepts should be encouraged. While success is important, children should also be challenged and encouraged to persist with tasks for increasing periods of time.

Asking children to talk about how they solved problems should help them learn from each other and come to realise that there is often more than one way to deal with a situation. Setting a single problem and providing ample time for solving and discussing it are likely to promote qualities of thoughtfulness and persistence.

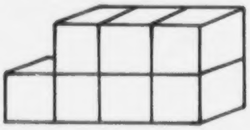
MATHEMATICAL EXPRESSION

Experiences with mathematical expression should be provided which enable children to:

A1 read and write numerical statements, and use and interpret pictorial representations

Possible activities:

- Match objects with pictures or diagrams of the objects (e.g. match a cylinder, cone, sphere correctly with pictures).
- Given a diagram, produce a three-dimensional model (e.g. give children a picture of a stack of cubes and have them produce the stack).



- Devise shorthand ways of representing the results of combining, separating, grouping and sharing collections.
- Translate between symbolically and orally expressed statements (e.g. the sum of four and three is seven can be written as 4 + 3 = 7).
- Explain to a peer the meaning of a question from mathematics text material (e.g. in the instruction 'complete: 7 + [ ] = 10', the question is 'what number adds to seven to give ten?').
- Reconstruct an operation on concrete materials from a picture or diagram drawn by a peer, and discuss the difficulties involved.

A2 clarify, use and interpret mathematical terms and phrases

Possible activities:

- Discuss everyday meanings of words and compare with mathematical meanings (e.g. table).
- Identify mathematical terms and phrases which children do not understand the meaning of (e.g. I don't know what 'group the counters' means).
- Frame questions to ask for clarification or to show areas of misunderstanding (e.g. I don't think you can take 8 away from 6 because when I punched 6 – 8 on the calculator it gave me a 2 with a 'take away' sign in front of it; what does that mean?).
- Compare and use comparative expressions which are very close in meaning (e.g. each of the following means 'approximately but known to be less': almost, nearly, not quite, just under, a bit under, a little under, a bit less than).
- Compare and use bipolar comparatives in order to clarify meaning (e.g. distinguish big–small, from tall–short).
- Make personal use of mathematical phrases to describe objects (e.g. these shapes all have three sides; this is one lot of 3, two lots of 3, three lots of three, ...) and actions on objects (e.g. I cut the apple into four equal pieces so each piece was a quarter of the apple).

ORDER AND ARRANGEMENT

Experiences in order and arrangement should be provided which enable children to:

A3 identify and describe regularities and differences

Possible activities:

- Sort and classify objects within their environment according to one or more criteria generated by self and by others.
- Describe attributes used as the basis for sorting and classification.
- Explore a range of decorative and functional patterns from children's various home cultures, such as in daily rituals, designs on clothing, artefacts, folk dancing and holidays.
- Create, describe and extend patterns using a wide range of resources such as objects, sounds, rhythm and movement.

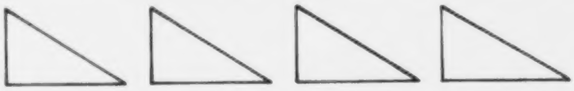
A4 recognise, produce and use spatial and numerical patterns

Possible activities:

- Make a number of different patterns with the same materials.
- Recognise, make and describe the same pattern in different materials.

○	○	●	○	○	●	○	○
clap	clap	jump	clap	clap	jump	clap	clap
4	4	7	4	4	7	4	4

- Recognise, copy, continue and devise simple repeating patterns of numbers and represent these patterns in spatial designs.
- Recognise and use spatial patterns (e.g. I can fit these triangles



together by turning every second triangle upside down to make rectangles then fitting the rectangles together).

- Explain conjectures (e.g. we thought that if we made different-shaped rectangles but used 12 tiles each time, the rectangles would each be the same length around the outside).

JUSTIFICATION

Experiences with justification should be provided which enable children to:

A5 explain results, conjectures and guesses in everyday language

Possible activities:

- Explain how each element of a number pattern can be obtained from the previous element (e.g. my pattern is to begin with one and add two more each time).
- Explain the basic pattern in our numeral system (e.g. Sixteen sticks can be made into one bundle of 10 and 6 over, and that is why it is written as 16. Using this rule, I think that 35 sticks could be bundled into three bundles of 10 and 5 over.)
- Use everyday language to explain how each element of a spatial pattern can be obtained from the previous element (e.g. we made stairs with squares: the first stair had two rows and that took three squares; the next stair had three rows and that took six squares ...; each time we added one more square than the time before).
- Test conjectures (e.g. we made different shaped rectangles each from 12 tiles and found that they didn't all have the same length around the outside).

PROBLEM-SOLVING STRATEGIES

Experiences with problem-solving strategies should be provided which enable children to:

A6 use problem-solving strategies and compare strategies for solving the same problem

Possible activities:

- Solve mathematical problems which encourage the use of such strategies as:
  guessing, checking and improving;
  acting out a problem;
  looking for patterns;
  working backwards;
  making a model or using objects;
  drawing a picture;
  making a list.
- Discuss the 'errors' and 'wrong' ideas which have been helpful in reaching a final solution.
- Discuss and compare strategies, reflecting on their usefulness in particular contexts.

A7 clarify mathematical questions and pose related questions

Possible activities:

- Clarify ambiguous tasks such as 'order the jars from largest to smallest'.
- Finish one problem by posing another (e.g. I ordered the jars by height; will I get the same result if I order them by how much water they hold?).
- Provided with a simple context (e.g. a picture of a table set for dinner) pose questions about the situation for self or others to solve.

A8 undertake linked sequences of problems, individually and collaboratively

Possible activities:

- Carry out structured investigations in number (e.g. find some addition combinations that make 11) with simple extensions (e.g. do we have them all? how can we tell? can we think of another question like this?).
- Carry out structured investigations in space and measurement (e.g. make a rectangle with 12 squares; make another; how many are there?) with students extending in some way (e.g. what about 9 squares? …).
- Carry out open-ended explorations in number (e.g. find patterns on the 'one hundred chart').

MATHEMATICAL INQUIRY in BAND B

Overview

Children should continue the informal study of patterns, although the structured spatial and number patterns explored should increase in sophistication. They should talk and write about the patterns they observe, developing the language with which to express their ideas and explain and justify their conclusions.

Making conjectures and and trying to validate or invalidate them should be a focus of activity in this strand. Children should test their conjecture by trying a range of cases and searching for exceptions. This may be done collaboratively (e.g. A boy may tell his partner his conjectured rule, and she may help him test it by asking: Does it work for squares? Does it work for …) and individually (e.g. My rule worked for 2, 3, 5 and 1 but would it work for bigger numbers like 20? What about fractions: should I try them?) During band B, the notion of a generalisation being 'always true' should begin to crystallise and children may offer general arguments about why a rule 'must be so'.

Children should be given the time to reflect upon the strategies employed to tackle problems. It is through reflection prior to solving a problem (Have I seen a problem like this before?) and afterwards (Now I have solved this problem, what does it tell me about other problems?) that problem-solving competence is developed.

Long periods of self-sustained mathematical activity should not be expected during band B, but children should be encouraged to persist with problems and to ask other questions about the situation. Having children work together on problems and posing questions for each other can be useful in helping them to sustain an investigation. Whole-class investigations of mathematical situations are also important and can be useful in providing a structure for children to follow when engaged in their own investigations. They should focus explicitly on the strategic skills they need in order to organise and manage their way through problems, either individually or collaboratively.

Working individually or in groups, children should give short written and oral reports of their investigations to other class members:

> We put red counters on the multiples of 5 and yellow counters on the multiples of 3. Only multiples of 5 x 3 (15) were covered by red and yellow. This worked for 4 and 3, and for 4 and 5. We thought it might always.
>
> Then we tried to find some numbers it didn't work for and found 4 and 6 don't work because 12 gets covered. We then thought it might only work for primes. We found that it did work for all the pairs of primes we tried but it also worked for other pairs like 4 and 3, 8 and 9, 5 and 6. We are now wondering if there is anything special about those pairs or if there is no pattern.

Their command of mathematical expression should develop considerably during these years. In order for it to do so, however, they need ample opportunity to use the mathematical language for themselves. They also need to be encouraged and helped to use the language with increasing clarity and precision so that it can support their own learning. For example, while technical geometric language is not expected, being able to use conventional spatial language (see **Space**) can assist children in explaining and remembering what they have observed about shapes and movements.

MATHEMATICAL EXPRESSION

Experiences with mathematical expression should be provided which enable children to:

B1 draw diagrams and read and write mathematics using simple formulae, pictures, tables and statistical graphs

Possible activities:

- Relate symbolic mathematical expressions to their linguistic form (e.g. % is 'per cent', > is 'greater than') and read aloud sentences expressed at least partly in mathematical symbols (e.g. 20% of \$300 = \$60, 2.35 x 10 = 23.5).
- Devise shorthand ways to write oral statements (e.g. 3 eighths + 5 eighths = 8 eighths).
- Write symbolic statements to match oral statements (e.g. write in mathematical symbols, 'one-half of one-quarter is one-eighth').
- Compare various usages of the point (.) in Australia and in the countries of origin of children in the class.
- Draw two-dimensional sketches of three-dimensional shapes made with sticks or straws.

B2 clarify, use and interpret mathematical expressions and phrases

Possible activities:

- Explain the various uses of words which may have different meanings within and outside mathematics (e.g. figure, cone, area, multiply, group).
- Identify the source of misunderstanding or confusion in a mathematical statement and ask for assistance (e.g. I think my problem is that I don't know what they mean by 'factor').
- Ask for clarification about the meaning of an expression or phrase (e.g. what do they mean by 'reverse the digits'?).
- Write questions for peers to answer and discuss ambiguities present in the questions.
- Explain phrases in word problems and express in shorthand form (e.g. Mai is six years older than Kim: this means that if you took Kim's age and added 6 you would have Mai's age — so Kim's age + 6 = Mai's age.)
- Relate metric measures to each other using metric prefixes.
- Make personal use of conventional geometric language to describe shapes (e.g. all of these shapes have one curved surface and three flat surfaces) and movements (I rotated the design through a quarter turn).

ORDER AND ARRANGEMENT

Experiences in order and arrangement should be provided which enable children to:

B3 observe, represent and extend spatial and number patterns

Possible activities:

- **Identify, describe and continue spatial patterns based on symmetry, simple movements, and sequences such as . ∴ ⁙ ...**
- Create a spatial pattern according to a specified rule, generated by self or others. (e.g. make a pattern by turning your shape a quarter turn each time).
- Use computer graphics packages to create patterns based on a variety of movements.
- Develop general rules to help with mental arithmetic, e.g.:

  7 + 6 = 13, so 17 + 6 = 10 + 7 + 6 = 10 + 13 = 23
- Investigate numbers for divisibility by 2, 3, 4, 5, 9, 10 and 11.
- Conjecture in order to predict elements in number patterns, e.g. use a calculator to investigate:

  1 x 2 x 3 x 4 + 1 =

  2 x 3 x 4 x 5 + 1 =

  3 x 4 x 5 x 6 + 1 =
- Investigate relationships (e.g. between the number of sides in a regular polygon and the interior angle) and use them to make predictions (e.g. the interior angles of other regular polygons).
- Generalise a problem solution in order to make predictions about related problems (e.g. having found the number of diagonals for a pentagon, hexagon, heptagon, predict for an octagon).

JUSTIFICATION

Experiences with justification should be provided which enable children to:

B4 test conjectures in space and number and search for counter-examples

Possible activities:

- Test conjectures about patterns in number and space by testing further elements in a sequence.
- Search for counter-examples to a conjecture.
- Use an attempt at justification to refine a conjecture (e.g. we then thought our rule would always work for prime numbers).
- Test conjectures by systematically checking all cases (e.g. check all the rectangles that can be made with 36 squares to demonstrate that the square has the smallest perimeter).
- Predict the effect of a change in the value of a variable in a LOGO program and test the prediction by running the program.

B5 explain conjectures and results, using appropriate mathematical language

Possible activities:

- Explain simple number sequences verbally and predict elements (e.g. 3, 6, 9, ..., ..., or 251, ..., 273, ..., 295, 306)
- Play one- and two-stage 'guess my rule' games and describe the rule (e.g. whatever number I said, you added 1 and then multiplied by 2).
- Express orally, and in writing, relationships between shapes and begin to use simple 'if ... then' statements (e.g. if a figure is square then it is also a rectangle, but the reverse is not true).
- Express processes of conjecture and justification orally or in writing.

PROBLEM-SOLVING STRATEGIES

Experiences with problem-solving strategies should be provided to enable children to:

B6 use a range of problem-solving strategies, and compare and suggest alternative strategies for the same problem

Possible activities:

- Solve a range of mathematical problems which reinforce strategies of the kind indicated in band A and extend repertoire to include: making an organised list or table; restating the problem; separating out irrelevant information; identifying subtasks and attempting the first.
- Reflect upon and discuss different strategies applied to the same problem, comparing successful strategies and noting unsuccessful strategies.

B7 refine and clarify mathematical questions and pose related questions

Possible activities:

- Clarify the domain of interest in a given situation (e.g. when you ask us to find pairs of numbers that add to 10[1] can we use fractions?)
- Clarify ambiguous tasks such as 'find all the different pentominoes'.
- Given an open question about a situation (e.g. which is the best buy?), refine the question (by 'best buy' do we mean the cheapest or should we take other things into account?).
- Given a computational exercise pose questions to which the computation is the solution.
- Prepare questions about a situation (e.g. given a store catalogue) for peers to answer.

B8 undertake structured investigations, individually and collaboratively

Possible activities:

- Carry out structured investigations in number (e.g. find a number under 100 that has an odd number of factors; find another, and another …) with students extending in some way (e.g. do I have them all? how do I know?).
- Investigate patterns in number leading to new concepts (e.g. use a calculator to explore patterns leading to ideas of recurring decimals).
- Carry out investigations across the strands (e.g. make a rectangle with 12 squares; make another; how many are there? what about 9 squares? try some other numbers).
- Discuss the results of investigations comparing methods used and false trails (e.g. patterns that 'didn't continue').
- Recount, orally or in writing, the results of investigations.

B9 use personal and group organisational skills to help in tackling mathematical problems

Possible activities:

- Use a range of hints consistent with students' maturity for 'getting started' on problems (e.g. Get your materials ready. Read the problem — really read it. Make a start on the problem — write something or draw something. Read the problem again and try to think differently. When you have a good idea, write yourself a message. Explain what you have done. Think: does your problem give you 'what if …?' ideas for new problems?).
- Develop hints for what to do when 'stuck'.
- Use simple procedural rules to enable groups to work effectively on the solution of problems.

MATHEMATICAL INQUIRY in BANDS C/D

Overview

As they proceed through the secondary years, students should increasingly be expected to integrate mathematical ideas and to undertake cycles of individual and group problem posing and solving. They should prepare and present oral or written summaries, explanations and reports in which they describe and justify their processes and conclusions, and their accounts should be mathematically sound and literate.

Overly pedantic attention to the technical language of mathematics is likely to be unproductive for most students. Nevertheless, the use of appropriate mathematical language is likely to support learning, and students should be assisted to use mathematical language for themselves both orally and in writing. Explicit attention should be paid to the various ways in which words may be used colloquially and in mathematics (e.g. function) and to the use of logical connectives (e.g. if … then, therefore). In order that students can take some responsibility for their own learning of mathematics, the skills needed to read mathematical text should receive careful attention.

The capacity of computers to undertake vast numbers of computations and exhaustive case analyses makes some empirical investigations relatively easy. There is a risk that this may mask essential mathematical relationships and ways of thinking, and lead to a neglect of the analytic side of mathematics. Solving problems simply by 'brute force' is unlikely to lead to mathematical thinking or to an understanding of the power of mathematical analysis.

Students will need considerable experience with the cycles of conjecture, explanation and justification which typify problem solving and the generation of new knowledge in mathematics. This should include the preparation of arguments to convince others that, not only does a pattern hold for all the cases tried, but that it *must* always hold. They should also distinguish between highly plausible experiments, such as approximating the roots of polynomials using computer graphics, and mathematical proof.

Students should develop a repertoire of standard techniques for confirming or refuting conjectures. These may include, for example:

- systematically checking all possible cases;
- reducing the conjecture to a known result;
- using algebra to handle general arguments;
- substituting a conjectured solution into an equation;
- finding a counter-example by attempting to find a justification;
- finding a counter-example by investigating special cases;
- showing that the conjecture is inconsistent with a known result.

They should also evaluate the validity of arguments by, for example:

- checking the result in special cases;
- looking for common errors such as misleading diagrams or the ignoring of special cases;

- providing full details if these are not included in the argument;
- determining the values of variables for which an argument is valid;
- investigating what other results follow by similar arguments and seeing what is wrong if these are false.

Students should recognise that proof is partly about plausible argument and partly about agreed assumptions. They should prepare arguments based on given sets of assumptions; that is, they should make deductions of the form 'If I know (or assume) this, then I also know this'.

As they progress through bands C and D, there should be an increasing expectation that students 'polish' their proofs and present them coherently and succinctly. Those intending to proceed to further studies in mathematics may benefit from experiences in formal methods of proof and of axiomatic reasoning in both algebraic and geometric contexts.

Students should have available a range of standard and personal strategies for getting started on problems. Reflection is essential in guiding and interpreting mathematical activity, and students should be encouraged to take this aspect of mathematical work seriously. For example, working collaboratively, they may brainstorm starting strategies and mathematical content which may be relevant to a particular task and then discuss why they favour one over the other. Having worked on problems individually or collaboratively, they should then reflect upon and discuss successful and unsuccessful mathematical strategies, and the mathematical ideas which proved important.

MATHEMATICAL EXPRESSION

Experiences with mathematical communication should be provided which enable students to:

CD1 draw diagrams and read and write mathematics using symbols, tables, pictures and graphs

Possible activities:

- Match words and phrases with mathematical expressions which use parentheses (e.g. say $(a + b)^2$ as 'a plus b all squared', rather than 'a plus b squared').
- Express symbolic expressions orally (e.g. express $f(x) = 2x + 1$).
- Translate phrases in word problems into symbolic statements (e.g. the number 'seven less than n' could be written as $n - 7$).
- Read short pieces of mathematical text (e.g. an explanation of the meaning of 'prime number') and explain the meaning to peers.
- Write short pieces of grammatically correct mathematical argument.
- Given a qualitative graph of a relationship (e.g. improvement in guitar playing over a period of time) explain what the graph shows.
- Read in order to find out about some mathematics needed in problem solving (e.g. read a text which explains how to operate with indices needed for a problem about a bouncing ball).
- Read in order to find the meaning for unfamiliar expressions in mathematics (e.g. students may know of the rules for exponents but not heard them described as index laws).

CD2 use and interpret mathematical terms, expressions and phrases orally and in writing

Possible activities:

- Work collaboratively to explain the meaning of mathematical terms and expressions (e.g. hypotenuse, if and only if), redrafting to remove ambiguity.
- Explain differences in colloquial and mathematical meanings of words (e.g. function, proper, negative, satisfy).
- Interpret prepositions (by, with, from) in mathematical expressions (e.g. decide whether the question 'by how much does the number x exceed the number y?' can be answered by the operation of subtracting y from x).
- Work in groups to explain standard word problems to each other, 'saying it another way'.
- Describe a diagram to another student so that she/he can draw it unseen.

ORDER AND ARRANGEMENT

Experiences in order and arrangement should be provided which enable students to:

CD3 observe and represent spatial and numerical patterns and identify the same pattern in different contexts

Possible activities:

- Develop simple strategies, such as finding common differences, for determining the rule for number patterns.
- Use a calculator to explore number patterns, given a starting place, e.g. explore and explain the following pattern:

$$6^2 - 5^2 =$$

$$56^2 - 45^2 =$$

$$556^2 - 445^2 =$$

- Use spreadsheets and function graphing calculators to investigate and make links between compound interest, geometric progressions and exponential functions.
- Generalise a method for tessellating any triangle or quadrilateral and demonstrate by tessellating any triangle or quadrilateral.
- Compare the coefficients of the binomial expansion, the pattern generated by Pascal's triangle and outcomes generated by ${}^nC_r$.
- Experiment with a mathematical situation (e.g. the number of handshakes at a party if everyone present shakes the hands of everyone else) and make conjectures about the relationships involved.

JUSTIFICATION

Experiences with justification should be provided which enable students to:

CD4 convince themselves about the validity of conjectures and revise, refine or extend conjectures

Possible activities:

- Classify colloquial statements (e.g. Australians from Italy like pasta) and mathematical statements (e.g. a shape with four sides of equal length is a square) as being always true, sometimes true or never true.
- Find counter-examples for incorrect mathematical generalisations (e.g. 'a square is any shape with four equal sides', 'it doesn't matter in what order we multiply matrices').
- Investigate the truth or otherwise of numerical (e.g. $6 > 3$ or $|-7| > 5$) and algebraic (e.g. $a > 3$ or $|x| \geq x$, for all x) inequalities.
- Describe the conditions under which a mathematical statement (e.g. dividing one number by another gives a smaller number) will be true and not true.
- Investigate necessary and sufficient conditions (e.g. a square is a quadrilateral that ...; a square is a parallelogram that ...; a square is a rhombus that ...).
- Choose and use standard techniques (see overview on page 49) to confirm or refute conjectures.
- Investigate particular cases (e.g. does it work for zero?) to assist in formulating a justification.
- Use counter-examples to refine conjectures (e.g. the conjecture now becomes, for all $x \neq 0$, ...).
- Use a justification to refine or extend a conjecture (e.g. having proved that $\sqrt{2}$ is not rational, extend the conjecture to state that the square root of any prime is not rational).

CD5 evaluate the validity of arguments designed to convince others of the truth of propositions

Possible activities:

- Check the work of peers to help identify errors of logic (e.g. explain what is wrong with the following: $x(x-2) = 0, \therefore x = 0$).
- Analyse short chains of verbal statements and decide whether each statement follows from the preceding statements.
- Distinguish between the information and steps that are essential in an argument and those which are superfluous or illustrative.
- Analyse a short chain of verbal statements in which the conclusion is 'true' but does not follow from the previous statement.
- Check the proofs of peers, choosing and using appropriate standard techniques for doing so.
- Report on errors of reasoning or implicit assumptions in an unvalidated proof which invalidate it.
- Compare proofs for generalisations (e.g. compare proofs of Pythagoras' theorem based on different analytic techniques; see **Space**, D2).

CD6 construct arguments designed to convince others of the truth of mathematical propositions

Possible activities:

- Explain why a solution to a practical problem must work in general (e.g. asked to assist a gardener find the centre of the circular garden bed, explain how to, and why the method works).
- Express general arguments about properties of figures (e.g. if you cut a tessellating figure along a line of symmetry, the half figure will also tessellate because you can …).
- Prepare short chains of statements which lead logically from assumptions to new propositions (e.g. use congruence conditions for triangles as axioms in order to establish the congruence of two triangles).
- Draft and redraft a proof of a proposition until convinced it will withstand the scrutiny of peers; submit to the scrutiny of peers.

CD7 recognise the effect of assumptions on the conclusions that can be reached

Possible activities:

- From a small set of assumptions, work collaboratively to generate and justify as many results as possible (i.e. if we know these things, then we also know all of these things).
- Compare features of two systems given rules for their composition (e.g. from two different sets of conditions for membership of a committee, make statements about the consequences of each for the possible composition of the committee).
- Investigate the effect of changing one assumption on the conclusions which may be reached.
- Identify implicit assumptions inherent in problems (e.g. given 6 matches, make 4 triangles).
- Investigate the assumptions implicit in proofs of conjectures in order to refine conjectures (e.g. clarify the type of polyhedra to which Euler's formula applies).
- Compare the features of two simple axiomatic systems (e.g. two 'taxi' geometries with different definitions of 'shortest route') and find results consistent within each system.
- Undertake a historical study of the development of ideas about consistency and completeness of the set of axioms for Euclidean geometry.

PROBLEM-SOLVING STRATEGIES

Experiences in problem-solving strategies should be provided which enable students to:

CD8 use a range of strategies to solve problems, compare different strategies and suggest alternative strategies for the same problem

Possible activities:

- Solve a range of mathematical problems which reinforce strategies of the kind indicated in bands A and B, and extend the repertoire to include:

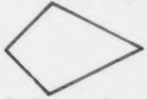
  solving a simpler version of the problem first;
  eliminating possibilities;
  trying extreme cases.

- In groups, brainstorm possible strategies for solving a problem and discuss which might be best to try first.
- Explain the strategy used for a problem and express it in general terms (that is, *name* the strategy) to enable it to be applied to new tasks (e.g. I tried to solve the problem for a smaller number of doors and that helped me see what was happening: basically what I did was to solve a simpler version of the problem first; I will remember that in future).
- Discuss different strategies applied to the same problem, comparing successful strategies and noting unsuccessful strategies.

CD9 pose, refine and clarify problems within mathematics

Possible activities:

- Given a geometric shape pose questions about it, e.g. ask as many different questions as you can about the following shape:

  Possibilities include: how many diagonals does it have? how many regions are formed if all the diagonals are drawn? what is the length of each diagonal?

- Pose questions about student-generated mathematical contexts for self or others to solve.
- As one problem is solved pose a related problem.
- Given a starting situation (e.g. a spiral on squared paper) brainstorm questions about it and choose some questions to investigate.

CD10 undertake open-ended mathematical investigations, individually and collaboratively

Possible activities:

- Find alternative strategies for solving a 'problem' set by the teacher.
- Undertake short investigations, the questions for which arise incidentally during classroom activities (e.g. in considering the definition of a prime, a student might ask what is special about numbers that have exactly two divisors; this might lead to a study of those with three).
- Carry out mathematical investigations which involve cycles of observation, conjecture and justification on topics of interest to students.
- Report orally or in writing on the results of investigations, making explicit the questions attempted and their origins, what and how data were collected, conjectures made, tests of conjectures and possible extensions.

CD11 develop personal and group organisational skills for tackling mathematical situations

Possible activities:

- Use a range of hints consistent with maturity for 'getting started' and 'keeping going' on problems (e.g. structuring work by: understanding the problem; devising a plan; carrying out the plan; and checking or reviewing).
- Develop ~~hints for~~ what to do when 'stuck'.
- Develop simple procedural rules to enable all group members to contribute in problem posing and solving situations.
- Make decisions about when group members should work on the same task and when they should take different parts of a task.
- Work methodically and review progress, individually and collaboratively.

Choosing and using mathematics

Purpose and scope

This strand addresses the mathematical processes involved in applying mathematics to physical and social phenomena. These processes involve moving between what are sometimes called the 'real' and 'mathematical' worlds. The processes emphasised in **Choosing and using mathematics**, like those in **Mathematical inquiry**, cannot be developed in isolation from the other strands. *They should pervade the whole curriculum throughout all the years of schooling.*

The strand is described under two subheadings: *Applying mathematics* and *Mathematical modelling*.

Applying mathematics

All students should be involved in applying mathematics to practical problems. This requires that they be able to:

- recognise that mathematics may usefully be applied to the problem;
- choose the most appropriate mathematical ideas or procedures from those available;
- use conventional notation and symbols and make decisions about the level of precision and accuracy needed;
- do the mathematics;
- interpret the results of using mathematics;
- judge the appropriateness of the methods used.

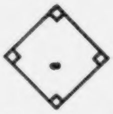
The opportunity to engage in these activities should be a major part of the work in the **Space, Number, Measurement, Chance and data** and **Algebra** strands.

Some circumstances occur so commonly that standard mathematical applications have been developed to deal with them; these are worth learning in their own right. Counting the number of people for dinner, estimating the discount on an appliance, deciding how much paint is needed for a bedroom and interpreting a scale drawing of the reticulation system are examples of such situations. The techniques may be so automated that people do not notice that they are doing mathematics. They may even forget that they ever had to learn the mathematics involved and did so successfully. In fact, a considerable part of school mathematics, particularly during the compulsory years, is directed at students acquiring standard mathematical techniques which everyone should be able to apply with ease.

During the post-compulsory years, techniques may be considered important for specific target groups of students. For example, learning to calculate equity mortgage valuations or distances and speeds from accelerations, or to convert to hexadecimal may be important skills for some students but less so for others.

If particular mathematics is considered to be sufficiently useful that students should be able to use it routinely, then it is important that they learn it in ways that maximise its usefulness to them. They should experience the range of contexts in which the mathematics is applied. For example, division should be applied to a wide range of tasks: deciding how many weeks it will take to earn a certain amount of money, how far one can travel on a tank of petrol, the proportion of people in the room who own a pet, and currency conversion. Also, they should have ample opportunity to choose which mathematics to use in situations in which external features do not provide artificial clues (e.g. When the whole set of exercises involve profit and loss, students do not need to choose what mathematics to use).

In addition to routine applications of mathematics, students need to be able to tackle situations about which they have not been taught and for which standard procedures do not immediately or obviously apply. For many non-routine problems, standard procedures may used if the problem is appropriately represented. A person may wish to mark out a softball diamond with 25 m between bases and know that it can be done efficiently if home and second base are marked first. This means the distance between them must be found.



In this case, seeing the softball diamond as a square and the line between home and second as its diagonal is the key to recognising that Pythagoras' theorem can readily be applied.

Some problems may not be obviously or necessarily mathematical and yet may benefit from the application of mathematics (e.g. scheduling a series of events, designing a storage system or publishing a school magazine). Also, mathematics often assists us to successfully read newspapers, information leaflets and reference books, or to listen to a documentary, news broadcast or the evidence presented in a trial. Terms which are not obviously mathematical but actually have precise mathematical meanings (e.g. inflation, acceleration, enlargement, mortality and fertility) may be embedded in verbal messages. Interpreting such messages often requires a careful analysis of these precise meanings.

Importantly, students need to learn to ask 'Will mathematics help here?'. If students are to choose to use mathematics in this way, they will need to experience its use beyond the mathematics classroom. This will, of course, require the involvement of teachers across the curriculum.

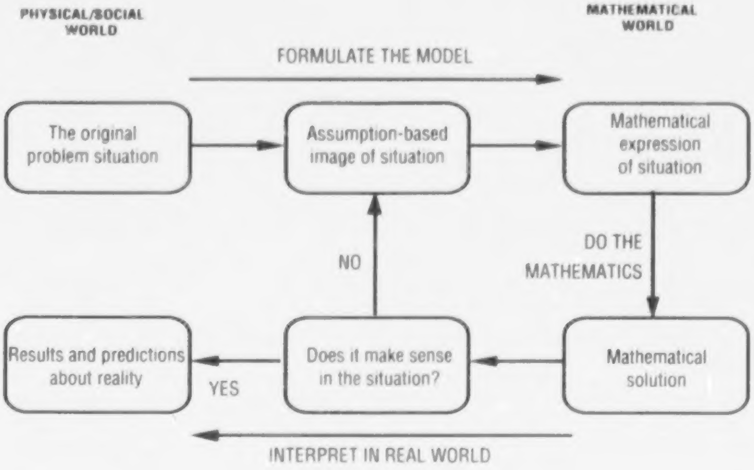
Mathematical modelling

A model is a simplified representation of an object, action, event or relationship which shares some but not all of the features of the original. Which features the model shares with the original depends upon the purpose of the model. For example, a model aeroplane will share some but not all of the features of the actual aeroplane. If the purpose of the model is to test for wind resistance, then the shape and structure of the model will be vital, as will the material from which it is made. On the other hand, if the purpose of the model is to consider passenger comfort, it may be made of plastic but match the internal dimensions and decorations of the actual plane closely.

A mathematical model is a mathematical representation of a situation. In creating a mathematical model, whether of the economy, the greenhouse effect or success in tertiary education, assumptions and simplifications have to be made. Which assumptions are appropriate depends on the situation being modelled. For example, the earth can be modelled in a range of ways depending upon the problem:

PROBLEM CONTEXT	MODEL OF EARTH
motion of planets	point
ellipses	disc
air flights between countries	sphere
car trip from city to city	plane
design a dam	none of above

Thus, the usefulness of a mathematical model depends on the assumptions upon which it is based. The essential features of mathematical modelling are suggested in the following diagram.


PHYSICAL/SOCIAL WORLD
MATHEMATICAL WORLD
FORMULATE THE MODEL
The original problem situation
Assumption-based image of situation
Mathematical expression of situation
NO
DO THE MATHEMATICS
Results and predictions about reality
YES
Does it make sense in the situation?
Mathematical solution
INTERPRET IN REAL WORLD

Formulating a mathematical model involves producing a simplified (assumption based) image of the actual situation which can be represented in mathematical terms (e.g. using a table, introducing variables and writing a formula, sketching a graph). We verify the model by using it to make predictions about the situation and checking the predictions by observation or experimentation. If the model is inadequate in some way, then we refine it or replace it with a more adequate one, cycling back through the processes.

Having decided that a model represents the actual situation 'well enough' for our purposes, the mathematical representation itself can be explored and manipulated in order to further investigate the situation and its possible behaviour under changed conditions. Also, models which have been developed and found useful in one context are quite frequently found useful for dealing with problems arising in quite different contexts.

Mathematical modelling is a powerful intellectual tool and is a major feature of modern physical, biological and social sciences. Knowing what models are and recognising when they are in use are two skills necessary for informed participation in society. Students should know that all applications of mathematics are based on mathematical models. In principle, applying mathematics requires that we make certain assumptions about the situation at hand and represent the relationships mathematically. In practice, we often do not make these processes explicit. If any or all of the assumptions upon which a model are based are suspect, so too may be the conclusions reached or justified by the use of that model. Those who misunderstand this and who believe that all uses of mathematics are equally valid may be confused by predictions of various kinds and vulnerable to exploitation.

In 'applying mathematics', students will engage in many of the key processes of mathematical modelling, albeit implicitly. The purpose of the activities described under 'mathematical modelling' is to make explicit the full modelling cycle. That is, students should learn about mathematical modelling and recognise and experience the key processes for themselves. This explicit emphasis on the full modelling cycle is suggested only for bands C and D.

CHOOSING and USING MATHEMATICS in BAND A

Overview

A common feature of the early years of schooling is that children construct mathematical ideas on the basis of their knowledge of their physical and social world. For example, the investigation of 'take away' situations in which both the problem and the solution strategy make sense in the children's world enables them to construct a meaning for subtraction. Applying mathematics involves movement in the other direction. We use mathematics to make better sense of some aspect of our physical or social world.

Children need to experience mathematical concepts and skills in a variety of contexts. In order to fully construct a meaning for subtraction, for example, young children will need to experience 'take away', 'difference' and 'comparison' situations and link these to a single mathematical idea. In order make full use of subtraction, they will need experience in choosing and using it in contexts which range over all the common uses and in settings in which no obvious cues are provided to indicate that subtraction is expected.

From the earliest years, children should be encouraged to think about the assumptions they need to make in order to use particular mathematical ideas or techniques. For example, they may collect data of the food likes and dislikes of class members (see **Chance and data**, A4 and A5) and make decisions about whether the numbers in various categories can reasonably be added (**Number**, A7). They should also decide how to interpet the result of performing a procedure in context (e.g. We made half of the recipe, but that doesn't mean we made half a cake — it means we made a cake which was half as big).

Children need considerable experience in representing 'real-world' problem situations in ways that enable them to be dealt with mathematically. Consider a situation in which children need to find the total amount of money owed for two purchases (costing, say, 27c and 35c), but do not have a standard way of dealing with such situations. The children might represent the problem using concrete materials. This is not as simple or obvious a process as is sometimes assumed. It would require that they:

- recognise that we can focus on mathematical features of the money (i.e. number quality, 27 and 35) but ignore other features (e.g. colour);
- choose an appropriate representation (e.g. structured material such as Multibase Arithmetic Blocks (MAB) or toy money);
- correctly represent the two amounts of money;
- see that the action of combining the two lots (of MAB or toy money) matches the process of finding the total of the two amounts of money;
- carry out the operation of combining the lots and determine the result;
- interpret the result in the original context.

Children may have represented the problem in a range of other ways depending on the wording of the problem, the size of the numbers used, the physical materials and the mental representations available to them. Representing problems, whether with concrete materials, pictures, mental images or symbols, is a complex process. In this case, the MAB (toy money) have been used as transitional representations 'on the way' to the purely mathematical representation, 27 + 35.

Number work does not provide the only representational activities. For example, children may investigate what makes buildings stable by building with constructional material and why bicycle wheels are round by experimenting with various shapes as 'wheels'.

From their earliest years of schooling, young children should have the opportunity to use mathematics to solve problems of practical interest to themselves. These may range from deciding whether their spending money for an outing will stretch to a sweet to producing a book about themselves, and should draw on all aspects of the mathematics curriculum.

APPLYING MATHEMATICS

Experiences with applying mathematics should be provided which enable children to:

A1 use mathematics in dealing with practical or imagined situations from their experience

Possible activities:

- Develop mathematical concepts (e.g. area) in the context of problems (e.g. can I cut this paper to cover the box?).
- Apply mathematics incidentally to daily activities (e.g. how many are missing if there were 25 kits this morning and only 22 are here now? would you remind me when it is ten-thirty?).
- Investigate the price of two different-sized but similar items (e.g. which is the best buy, a pack of 10 crayons at 73c or a pack of 20 at $1.65?).
- Apply mathematics to practical situations which are not necessarily mathematical (e.g. in making a book about oneself, measure and collect, organise and interpret data, design pages and decide how many pages, how big, how to fold it).
- Find alternative methods for solving a problem (e.g. find half the paper clips in as many ways as possible).
- Explore open-ended practical situations (e.g. three children go mushrooming agreeing to pool and then share the mushrooms they find: how should they share them?).
- Investigate the mathematical ideas embedded in children's stories.

A2 represent problems using objects, pictures, symbolic statements or mental images

Possible activities:

- Refine or clarify problems (e.g. to organise the delivery of the messages I need to put them in order by the number of the classroom to which they have to go).
- Choose appropriate materials or pictures to represent problems (e.g. draw a plan of the school to assist in organising the route for message delivery).
- Identify situations in which addition, subtraction, multiplication and division are appropriate or inappropriate, restating problems mathematically (e.g. I have to find 6 lots of 13).
- Identify a wide range of practical problems which can be represented by a single arithmetic operation (e.g. know that the 'division' key on a calculator is appropriate for sharing and grouping problems which occur in a range of contexts).
- Use representations to test predictions (e.g. I think that in 30 minutes time it will be a quarter past ten).

A3 interpret solutions in the problem context and comment on results

Possible activities:

- Interpret solutions in the original context (e.g. if I buy the sweet I will still have 55c left, which will be enough for my fare).
- Check that solutions make sense in the original context (e.g. does it make sense that this box could hold more than that box?).
- Orally or in writing explain what a solution means in the context (e.g. two packets of 10 will cost 2 x 73c, which is $1.46, which is less than $1.65: this means that it would be cheaper to get two packets of 10 rather than one packet of 20).

- Comment on the assumptions necessary to use the solution strategy adopted (e.g. multiplying the price of one packet by two assumes that two packets of 10 is as good as one packet of 20, but then I would have only 10 different colours).
- Judge the appropriateness of a solution in a given real situation (e.g. I think we asked the wrong question: why would we want two sets of the same 10 colours?).

CHOOSING and USING MATHEMATICS in BAND B

Overview

During band B, children should use the full range of mathematical concepts, skills and notation available to them in a variety of practical situations. They should measure, estimate and calculate according to given sets of expectations about audience and accuracy.

Children should construct new mathematical ideas from a rich variety of familiar contexts. For example, many young children will initially develop a concept of multiplication as 'repeated addition'. This is only a partial concept of multiplication which does not include multiplying by common or decimal fractions (as is often needed for scale and area problems). Informal scale and area activities may be seen as a source of experience from which children can construct a more complete meaning for multiplication.

Investigating prices of different-sized similar items (e.g. chocolates) to determine the best buy *prior* to any formal teaching about ratio, provides a modelling activity of wide scope. It enables children to apply existing concepts and skills to a situation for which they have no immediate solution strategy. It can also provide a basis in experience for the concept of ratio.

Having developed particular mathematical ideas and procedures, children can apply them in new contexts. For example, having concluded that the area of a rectangle can be found by multiplying the length by the width (with a calculator), children may be confronted with the question: Find the area of your desk top.

They then have to decide whether the method is appropriate (e.g. Is the desk top a rectangle? Can we check? Is it close enough?). If the children decide that the formula may be used, they will need to measure to a desired level of accuracy, apply the formula and interpret the answer. If they decide a particular method should not be used they will need to explain why not and find an alternative (e.g. We can't use the formula here because it isn't really a rectangle. We'll find another way.)

Children will need the opportunity to use particular mathematics for tasks which range over all the common types. For example, they need ample experience of the use of multiplication to calculate costs and distances from rates, areas, the results of enlargements and reductions, combinatoric problems and simple conversions. The goal is to ensure that they are able to use multiplication almost automatically in all the commonly occurring situations.

Non-routine problems should vary from structured to relatively unstructured and from those which can be completed within a lesson to those which may take several sessions. Problems will usually arise in the physical or social world but they may also come, for example, from within a computer microworld or a piece of fiction. The important thing is that the problems attract and involve children and that children should have some ownership over the problem and its solution. 'Real' problems often have no established correct answers or solution strategies, and the test of the appropriateness of the solution is empirical (it works!). In this sense, children are as able as the teacher to judge whether they have solved the problem.

For at least some of the time, children should be free to take initiatives and work through problems from beginning to end, deciding what questions to ask, what data to collect, what action to take and how to verify the solution. More often than not, this will involve them in collaborative activity, in which they discuss and debate approaches and strategies, and cooperate in reaching conclusions. Such problems may also provide an excellent opportunity for children to bring together the various mathematical and non-mathematical skills they develop during the primary years.

APPLYING MATHEMATICS

Experiences with applying mathematics should be provided which enable children to:

B1 choose and use mathematical skills to make decisions

Possible activities:

- Develop mathematical concepts (e.g. concept of scale) in the context of problems of interest to them (e.g. how can we make a model of the house which 'looks right'?).
- Develop mathematical skills from problems of interest to them (e.g. develop a calculator-based procedure for finding measurements for the scale model from the original measurements).
- Recognise situations in which particular mathematical representations (e.g. area formulae) are appropriate or inappropriate.
- Make decisions about cheapest, quickest, best, most likely, etc.
- Apply standard mathematical procedures (e.g. calculating quantities, given rates) in a wide variety of contexts.
- Use learned mathematical skills and concepts to deal with problem situations of interest to them (e.g. do you know anyone who has lived a million days? investigate and report).
- Use mathematics to help solve problems which are not necessarily mathematical (e.g. plan a doughnut-eating competition for the school fete).

B2 clarify and pose problems arising in practical or imagined contexts

Possible activities:

- Given a problem situation (e.g. there isn't enough space in the room to have a quiet reading section), ask questions of clarification (e.g. do we care? can we read on the verandah? are we allowed to rearrange the furniture?).
- Pose subtasks which may lead to resolution (e.g. can we think of a way of rearranging the furniture to leave enough room for the quiet reading section?).
- Pose mathematical questions about a problem situation (e.g. how much room do we need to leave between desks so people can move between them?).
- Given a context (e.g. a photograph of a car being driven through a town) pose mathematical questions about it (e.g. what season and time of day is it? will the shadows tell us in which direction the car is heading? about how tall is the man who is in front of the shop?).
- Check that there is sufficient information to solve a problem and, if not, ask questions to elicit the other information needed.

B3 represent practical problems using objects, diagrams, symbols or mental images

Possible activities:

- Use visual imagery to represent a problem situation (e.g. find the volume of the room) leading to the development of a strategy (e.g. we could build skeletons of a metre cube and fill the room ...; do we have to build all of them?).
- Represent essential features of problem situations visually (e.g. draw a plan of the room and cut out plans of the furniture to see how it might be rearranged).
- Model natural phenomena (e.g. model a beehive with hexagonal prisms in order to consider why bees build hexagonal cells).
- Investigate the use of mathematics in fiction or daily newspapers (e.g. were the Lilliputians correct in how much food to feed Gulliver?).

B4 verify and interpret solutions with respect to the original problem

Possible activities:

- Make judgements about whether the conclusions reached make sense in the situation (e.g. when I added the number of us who play tennis to the number who play hockey, I got more than there are in our class; this doesn't make sense, so I need to check what I did).
- Express results orally, in writing or physically.
- Test predictions which result from problem solving by observation or by experiment (e.g. I tried it out on a piece of paper and now I think that the tables will fit if we arrange them this way).
- Run LOGO programs to test predictions.

CHOOSING and USING MATHEMATICS in BANDS C/D

Overview

The same scope statements apply to bands C and D. However, the problems students deal with and the solutions they produce should develop considerably as they proceed through the secondary years. Students should demonstrate a substantial expansion of mathematical ideas and skills and, consequently, there should be an increase in the mathematical demands of the problems they investigate. They will also gain more experience of the world and of other curriculum areas, increasing the range of situations to which they can apply mathematics. Finally, as students mature there should be an increase in their capacity to undertake modelling projects independently of the teacher.

Students should recognise when standard mathematical techniques, such as calculating the discount on an item, are clearly appropriate and also recognise when such techniques lead to foolish answers (e.g. concluding that 3.75 eggs are needed for a recipe after having used a standard technique for solving proportions). They should be expected to choose and use appropriate mathematical techniques and tools by careful consideration of the circumstances. For example, they should decide how accurate they need to be in a particular context and, as a result, choose suitable measurement units and devices and calculation techniques. They should also be prepared to explain and justify their choices.

A useful activity will be to identify the models inherent within familiar applications. Exercises like this often appear in texts: If it takes me 50 minutes to walk 4 km around the lake, how long should it take me to walk 3 km to the shops?

It is generally expected that students will use a standard technique such as 'the unitary method' or 'setting up a proportion' to deal with the task. Students could make explicit the model implied with these solution strategies and consider under what circumstances the assumptions are reasonable. For example, would an assumption of direct proportion be reasonable if the distances were 0.5 km and 15 km or if the trip to the shop was a straightforward walk along a road while the route around the lake was through heavy scrub? Would the answer depend on how fit the person was? They might also investigate the general question of time taken for walks of different lengths and attempt to build an improved model.

If all that is required is a rough estimate of when the person will arrive at the shops, an assumption of direct proportionality may be perfectly adequate. In this case, a very accurate calculation would be both unnecessary and foolish; an answer of '30 or 40 minutes' would be sensible. If a walker was pacing her or himself through a race, the same assumptions may not be reasonable and a more sophisticated model may be needed to determine how long it should take the walker to reach certain markers.

Students should understand the more general point that different models make different assumptions and that the same physical or social situation may be modelled in different ways. For example, while we may know that banks actually calculate interest on mortgages discretely (e.g. daily, monthly, quarterly) we may approximate the situation using a (continuous) exponential function and, within certain limits, get very accurate predictions. Also, Newton's and Einstein's models of motion involve different sets of assumptions. Each is in use and, while Einstein's model appears to fit our universe better, Newton's model is somewhat simpler to use and, for a great many purposes, does very well. If a simple model serves our purposes as well as a more complicated one, then it is likely to be more efficient to use the simpler one.

Students should undertake projects which have their origins in the physical or social world, and which require them to formulate and test mathematical models. In doing so, they should occasionally engage in the full modelling cycle. In particular, they should:

- pose, clarify or refine the problem;
- formulate a mathematical model by making useful and simple assumptions about the situation, collecting any data needed and representing relevant relationships in mathematical terms (e.g. using a table, graph, formula or equation);
- do the necessary mathematics and obtain a result or prediction;
- validate the model by observation or experiment, recycling if necessary to improve the model;
- interpret the solution.

Many students undertaking work from band D will be introduced to the concepts of the calculus (**Measurement**). Notwithstanding the rather procedural view of calculus presented over the years, calculus provides a powerful mathematical model for dealing with a wide range of problems. For some problems the natural variables are continuous (e.g. velocity, acceleration) and a discrete approach would be quite limited. Problems involving tangents (e.g. focusing light, sound or radio waves by parabolic dishes) generally need a continuous approach for their solution. Also, for the determination of areas and volumes (e.g. reservoirs, drainage problems) elementary geometry is often ineffective. Modelling problems can motivate the learning of calculus, and students should be encouraged to compare problem solutions (e.g. to maximisation problems) which do and do not use the calculus.

Problems that involve students in aspects of the modelling process are strategically more demanding than those involving the straightforward application of standard models and students will need to develop their own strategic or managerial processes for tackling modelling problems. The *technical* demands of the mathematics should be lower while students develop some of these important mathematical processes and strategic skills.

APPLYING MATHEMATICS

Experiences in applying mathematics should be provided which enable students to:

CD1 *choose and use standard techniques and formulae*

Possible activities:

- Use standard mathematical techniques to deal with everyday tasks (e.g. those associated with financial affairs, household maintenance).
- Use mathematical techniques which are standard within specific contexts (e.g. within sport, surveying, health, finance).
- Apply standard mathematical techniques (e.g. set up and solve proportions) in a range of different contexts.
- Choose and use standard formulae appropriately (e.g. recognise situations where $v = u + at$ and $s = \frac{1}{2}at^2$ apply and use the formulae).
- Choose and use mathematics to help solve problems which are not solely mathematical (e.g. design a board game).

CD2 choose and use data collection and analysis tools (including technology) with due regard to the demands of the situation

Possible activities:

- Decide how accurate one needs to be in given situations, justifying decisions.
- Choose measuring equipment which meets the demands for precision.
- Devise strategies for checking calculations.
- Choose and use calculators and computer software, paying attention to graphical accuracy and the effect of rounding procedures.
- Make and explain choices about whether or not to use computer software in a given situation (e.g. consider convenience, assumptions of software).

CD3 *choose and use mathematical skills to assist in interpreting information from a variety of sources*

Possible activities:

- Explain orally or in writing what is indicated by graphs taken from magazines, newspapers, encyclopedias or text materials from other school subjects.
- Comment on the strengths and weaknesses of graphs, their presentations taken from magazines and newspapers and the interpretations provided for them.
- Read or watch reports relating to the social and physical sciences, identify where mathematics has been used and comment on the appropriateness and effectiveness of its use.

CD4 recognise, use and evaluate standard mathematical models

Possible activities:

- Given familiar arithmetic tasks (e.g. calculate the quantities needed to triple a recipe) identify the models inherent in standard methods.
- Identify approximations that have been made in order to simplify presentation of information (e.g. expressing Bankcard interest rates as 23% per annum, which only approximates the effective rate calculated on daily balances).
- State assumptions implicit within a mathematical model (e.g. in using statistics about average weekly earnings of males and females to calculate borrowing capacity

for home loans, we ignore the possibility that lending institutions discriminate against female borrowers).

- Consider the accuracy of the predictions of a model (e.g. if interest is compounded continuously, use anti-differentiation to evaluate an annuity and evaluate the conclusions).
- Discuss models which have come to assume the status of 'real-world' phenomena, such as IQ, inflation, tertiary entrance scores, strike rate, batting average, considering what reality these models are intended to represent.
- Compare probability models (e.g. compare various pre-election public opinion poll predictors with actual election voting patterns).
- Identify when smooth graphs have been drawn to represent uneven flows of events (e.g. using a straight line graph to represent the weight gain of a baby in the first year of life).
- Apply the calculus to situations which are modelled by the rate of change of a variable or variables in order to find the magnitude of the variable at a given state or time.

MATHEMATICAL MODELLING

Experiences with mathematical modelling should be provided which enable students to:

CD5 *pose or clarify and refine problems*
Possible activities:

- Define, focus or restate the constraints in a situation in order to clarify the nature of the problem.
- Given the mathematical expression of a relationship (e.g. a graphical or algebraic expression of a function), generate hypothetical situations to which the relationship is relevant.
- Check that there is sufficient information to solve a problem and, if not, indicate what other information is needed in order to determine a unique solution.
- For situations in which there is insufficient information to determine a unique solution, frame a more general solution which sets boundary conditions on the possible values some independent variable might assume.
- Work collaboratively and pool information to make order of magnitude estimates (e.g. Fermi problems: how many cups of coffee are drunk in Australia each year?).

CD6 *formulate simple mathematical models and test by observation or experiment*
Possible activities:

- Distinguish significant from insignificant factors in a problem situation (e.g. when dividing by 12 to convert from annual to monthly interest rates, recognise the importance of the compounding effect for borrowers and the insignificance of the differing lengths of months).
- Identify the relevant variables in a situation (e.g. in investigating traffic flow, list variables such as the the change time for traffic lights).
- Identify relationships in a situation (e.g. assume that heat loss is proportional to skin area and express algebraically).
- Make assumptions and sketch graphs to model a situation and improve the model after experimenting (e.g. water level in different containers under a dripping tap).
- Model natural phenomena (e.g. represent humans as cylinders or prisms).
- Formulate indicial (or exponential) models of continuous phenomena (e.g. model tea cooling).
- Investigate optimisation problems (e.g. decide the best shape of baggage to maximise the amount a traveller can take on a trip while staying within the regulation dimensions provided by a Qantas airline ticket).
- Simulate chance events to produce probability models and indicators of likely outcomes (e.g. using coins, dice, computer routines to produce tree diagrams and tables).
- Graph data and determine an appropriate mathematical model (e.g. repair costs related to vehicle age, cattle weight related to age) and use the model to make predictions.
- Write and refine short computer programs (e.g. in LOGO) to produce geometric shapes.
- Use a spreadsheet to model problems such as finance (e.g. inflationary effects).

CD7 adapt mathematical models to provide better fit or make different predictions for testing

Possible activities:

- Use a given model to make predictions and test them against real data (e.g. given a linear model in the form $f(x) = Ax + B$, for the number of weeks needed to earn $5000 at average weekly earnings for the period 1976 to 1982, test its predictive power for more recent years).
- Alter variables in algebraic models and test the predictive power of the new expression (e.g. in the model of weeks to earn $5000 above, try different values of A or B for the period since 1982).
- Investigate approximate formulae and rules of thumb so as to suggest better models (e.g. given that the effective interest rate is approximately twice the flat rate, use a spreadsheet to test the formula for various time periods and interest rates and try alternatives such as $E\% = 2.1 \times F\%$).
- Redraw graphs of physical events in response to changes in conditions (e.g. given a graph of vertical height against flight time for a plane journey, alter it to incorporate two hours spent in a holding pattern over Sydney).
- Explain how alternative assumptions will result in altered predictions (e.g. in extrapolating from a downward trend in record times for a sporting event, discuss the effects on the predictions made if the general use of body-building steroids were allowed).
- Compare different models as predictors (e.g. fit linear and non-linear curves by eye and compare the predictive power against more recent real data; compare models of the forms $f(x) = \frac{A}{x}$ and $g(x) = -x + B$ as predictors of the relationship between house prices and distance from the city centre).

CD8 interpret solutions in terms of the original context of the problem

Possible activities:

- Explain results obtained in everyday language.
- Make sense of solutions in context (e.g. explain a negative solution in a projectile problem).
- Use the context to choose which of the possible solutions to an equation are relevant (i.e. solving $f(x) = 0$ for x may produce two possible solutions, only one of which is meaningful in the situation) and explain the reason for the decision.
- Critically evaluate whether a solution makes sense in the context and explain reasoning.
- Decide whether the solution produced by application of a model suggests that the model needs to be improved.

CD9 develop a personal set of managerial procedures to be used in tackling modelling problems

Possible activities:

- Explain to other students the steps gone through personally in developing a mathematical model.
- Listen to others relate their story of strategies used in modelling.
- Develop a class wall poster showing a set of steps for modelling, use it and modify it over time in response to class decisions (e.g. a poster might begin with the following

steps: argue about a real situation; make a list of simplifying assumptions; strip the problem statement to its bare essentials; choose a mathematical strategy; test; adapt; retest; report conclusions).

Space

Purpose and scope

We use spatial ideas for a wide variety of practical tasks. We describe our surroundings, find our way around and mark out and construct living spaces. Spatial ideas are basic to the solution of many design problems: producing inexpensive but sturdy packaging, laying out a page for maximum appeal, arranging pattern pieces to minimise waste or developing the orbital engine.

The mathematical study of space is called geometry. Geometry has been central in the development of notions of abstraction, deductive reasoning, and of the relationship of mathematics to the 'truth' about our world. Thus geometrical concepts and results have considerable cultural and historical significance and wide applicability to human activity.

This strand is described under three subheadings: *Shape and structure; Transformation and symmetry;* and *Location and arrangement.*

Shape and structure

A primary goal should be to have students develop useful intuition and knowledge about shapes and their relationships. Students need to investigate objects in their environment, analyse their shapes and classify them according to various criteria. Both two- and three-dimensional shapes are abstractions from the physical world and students should be helped to recognise these shapes in the objects they see and handle. Eventually, students should be able to make statements about the properties of whole classes of shapes and the relationships between them.

Students should also understand shape, structure and function as basic to design through a range of practical and challenging problems. They may consider, for example, what it is about the sphere that makes it good for ball games, why animals and plants have evolved into their characteristic shapes, and why people may find certain shapes pleasing and others not. They may also use mathematics to assist them design practical and aesthetic objects which may range from patchwork quilts to toys to stage lighting.

Finally, visualisation skills should improve as students physically manipulate objects, section them, represent them in two dimensions, and explore the relationship between an object (such as a tetrapack for holding fruit juice) and its net (i.e. the opened out and flattened shape from which it is made). The productive use of computer-generated images requires good visualisation skills. This should be anticipated by classroom activities which involve visual representation and interpretation of objects and relationships. Computer generated images can also assist in the development of visualisation and students should have appropriate experiences with programming languages such as LOGO or computer-assisted design (CAD).

Transformation and symmetry

The notion of transformation has immediate everyday relevance and is also an important unifying one within mathematics. This strand is concerned with transformations in space.

Three transformations of particular interest are *translation*, *reflection* and *rotation*. Translating, reflecting or rotating an object leaves its shape and its size unchanged and each is associated with symmetry, central in such fields as crystallography and particle physics, and the basis of many designs (e.g. floor tiles, Rubik's cube).

The *enlargement* transformation (which also includes reduction) leaves shape unchanged but varies the size, an idea which is the basis of such technology as cameras and projectors, the production of scale drawings and models and our understanding of the behaviour of certain plants and animals.

Other transformations may distort shape and size. Familiar examples include the transformations needed to produce two-dimensional representations of three-dimensional space. Students should investigate how three-dimensional objects can be represented in drawings (e.g. plans and elevations, isometric drawings, perspective drawings, contour maps, Mercator and other geographic maps) and learn to use some of the conventions associated with these representations. In doing so, students should identify characteristics of the original object which are maintained in the representation and those which are distorted in some way. *Topological* transformations may distort shape, size, and even straightness of lines. Techniques associated with topological transformations are dealt with under *Location and arrangement*.

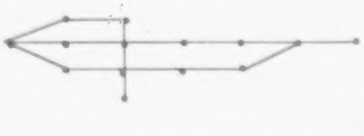
Students may develop and apply their understanding of transformation and symmetry across the curriculum: to art (e.g. heraldic designs, cultural/historical nature of the representation of depth, the work of Escher, surrealism), craft and technology (e.g. patchwork, knitting, model planes, and designing clothes, toys or desk accessories), science (e.g. crystal growth) or geography (e.g. map projections, contour and weather maps).

Note: While 'transformation' is another word for 'function' it is common in school mathematics to use the former term in geometry and the latter in algebra and calculus. On the other hand, when we reflect, rotate and translate functions, we speak of transformations of functions! These transformations provide many students with their first experience of composite functions.

Location and arrangement

A further dimension to the study of space is concerned with location and arrangement. The positional and directional concepts underlying location and its representation include familiar everyday concepts (e.g. down, on, under, here), and more technical concepts (e.g. angle, locus, parallel, perpendicular, reference point, grid, coordinate and graph). Students should study ways of representing location and arrangement and compare some of the conventions associated with such representations. They should use sketches of their locality or road maps to describe the position of local features, understand and use bearings to define direction, and specify location by using coordinate systems. As they progress through the secondary years, students should learn to use some of the more formal analytic techniques of coordinate geometry.

Notions such as boundary, path and region, interior, exterior and 'betweenness' underlie the mathematical study of problems which are about arrangement and in which measures, for example, of distance or angle are not central. Thus we may represent a rail system by a 'network' which shows the stations as dots with lines joining those which have a direct route between them.



The stations will be in the right order and be correctly connected but distances and directions will be ignored. The network can be thought of as a topological transformation of the stations and train routes. Students should develop elementary ideas about finite networks, solving simple route and network problems and undertaking informal investigations of how networks are used as models of such diverse areas as flood control systems, telephone exchanges, power grids, production schedules and nervous systems.

Spatial visualisation and equity

Having a developed ability to manipulate images in the mind (spatial visualisation) is useful. It improves our capacity to find our way around, to recognise shapes, arrangements and locations in different orientations and to imagine in advance what a shape or arrangement will look like. It is also directly applicable to a wide range of occupations (e.g. dressmaker, stage designer, landscape gardener, industrial designer, pilot and sculptor). Visualisation is of obvious importance in geometry and many find it a powerful tool in mathematics generally. Making use of visual or spatial imagery as a balance to verbal logical reasoning may be particularly supportive of learning for some Aboriginal and non-English-speaking-background (NESB) children.

Whether through play and 'tinkering' at home or through the study of subjects such as technical drawing, some students will have been in a better position than others to develop their spatial visualisation skills. Such students are more likely to be boys than girls. Boys are also more likely than girls to express confidence about their capabilities in these areas. It is important that this does not result in some students gaining greater access to materials and activities, perhaps leading to the inhibition of other students and to increased differences in confidence and competence. For equity reasons, the development of spatial visualisation skills should not be left to chance.

A note on language

In this statement, an informal convention of referring to two-dimensional shapes as figures and three-dimensional shapes as objects has been adopted. Also, technical language has been used where no everyday correct equivalent is available (e.g. transformation, reflection symmetry, three-dimensional, prism). However, *the formal teaching and testing of the vocabulary of space should not be the focus of work in this strand.*

SPACE in BAND A

Overview

During the early years of schooling the emphasis should be on exploration, both free and structured, of the children's local environment and objects within it. Young children should use a wide variety of material for making things, and for producing models of familiar parts of their environment. They should handle objects and observe their properties, describing them in their own language (e.g. This box has eight corners. Its sides are all squares.)

Young children should carry out changes to the shape, size or position of objects and observe the effects of these changes (e.g.When I turned the square jigsaw piece around, it still fitted into the same space). They should also note and describe their body movements in dance, drama and play, and the movements of objects around them, leading to an informal understanding of different transformations.

Rearranging and fitting together plane shapes, and slicing, stacking and packing solids provide important foundational experiences for the measurement concepts of area and volume. The links between the strands is very evident in these activities.

An intuitive feeling for scale should be developed through integrated activities in number, space and measurement. For example, children may play games requiring them to match component parts according to a rough scale (e.g. This chair is for the baby bear), and later, build scenes and draw plans to an intuitive scale, asking 'Does it look right?'. During these years, children should learn to recognise a range of representations of environmental objects in photographs, paintings, drawings and diagrams.

Basic ideas of location can be developed through everyday activities and games. Young children should arrange and rearrange familiar objects (from toys to furniture to the contents of their tray) both freely and by following oral directions. Children should also give and follow directions to find their way around an environment, including a computer environment. It is important to note that children's 'local environment' may be high-density inner city, a small rural town, a dairy farm or a small island community, and that, within these categories, their homes and lifestyles will vary. Standard text materials are often not helpful in this regard.

Some young children may come to school with a capacity to handle directional and spatial language which is exceptionally good by comparison with other children. Indeed, in some traditionally oriented Aboriginal communities, children's mastery of directional and spatial relationships is taken as a sign of intellectual prowess in much the same way that counting skills are in mainstream Australian society. These spatial strengths should be encouraged and used as starting points for other mathematical ideas. On the other hand, some students may have had very little experience in exploring their environment or in representing and describing that exploration. Such students will require a great deal of structured experience if they are to develop these skills.

Children should develop spatial language in much the same way as they learn to talk about various animals and objects — by hearing it used appropriately by others (their teacher and peers) and being encouraged to use progressively more sophisticated language in describing their experiences. This is important for all children, but particularly for children from families in which English is not the first language spoken in their home. During these years, most children will distinguish triangles, squares, circles, rectangles, cubes, spheres, cylinders and cones and use the everyday language of position such as up, down, across and under. The testing of mathematical vocabulary, as such, is inappropriate.

SHAPE AND STRUCTURE

Experiences in shape and structure should be provided which enable children to:

A1 build structures and make and investigate geometric models

Possible activities:

- Build structures from component shapes freely and according to criteria provided (e.g. make the tallest building you can with these blocks) and discuss (e.g. these pieces were not useful for the building because ...).
- Use a variety of materials to make familiar objects (e.g. a ball, house or dog) and describe features (e.g. the ball rolls, the house has a high roof and walls like rectangles, the dog has four legs).
- Slice common objects (e.g. oranges, carrots) and models (e.g. from plasticine or foam) to investigate different cross-sections, and use thin slices or prints of sections to record results.
- Solve simple design problems (e.g. use this piece of paper to make a box without a lid).

A2 compare and classify objects and describe in everyday spatial language

Possible activities:

- Compare objects and describe differences and likenesses using spatial language (e.g. round, smooth, curved, rolls, like a box, edge, face, base).
- Sort and classify objects in planned ways or according to criteria (including shape, dynamics and function) generated by others.
- Analyse the spatial features of geometric models (e.g. this cube has six faces, all squares; do all boxes have the same number of faces?).
- Identify or describe features of an object, having felt but not seen it (e.g. a child places a toy in the hand of another whose eyes are closed).
- Investigate the shapes of components of familiar rooms, houses and buildings and discuss advantages and disadvantages of the use of the shape (e.g. why do you think it is this shape? could it be any other shape? what if it was round?).
- Recognise and discuss aesthetic qualities of shapes in the environment (e.g. notice the differences between this building and that one; which do you prefer? why?).

A3 make and draw a variety of plane figures

Possible activities:

- Use a variety of techniques (e.g. cutting, tearing, pasting, making prints, geoboard, and drawing or with templates and tracings) to produce shapes.
- Use templates to construct complex pictures freely and according to specifications (e.g. can you make a picture of a bird from these shapes?) and describe the components and the end product.
- Use 'turtle' graphics or an equivalent to construct computer pictures.
- Cut boxes (e.g. toothpaste, Toblerone™) to form nets of objects and compare different nets of the same object.
- Predict the effect of variations on familiar designs (e.g. pretend you are riding a bike with square wheels; show me what it would be like).
- Give and interpret oral directions for producing a complex figure.

A4 compare and classify figures and describe in everyday spatial language

Possible activities:

- Use the vocabulary needed to describe spatial features of figures and templates (e.g. curved, side, straight, corners, circular, triangular).
- Sort and classify plane shapes according to your own criteria or criteria generated by others.
- Select a simple plane shape on the basis of a description of its spatial features (e.g. it has four sides and they are all the same length).
- Describe the spatial features of plane shapes including those identified in the environment (e.g. some of the corners are more pointy than others) and figures and templates (e.g. this rectangle has four straight sides and four corners and the corners are square corners).
- Identify plane shapes embedded in other objects (e.g. the top of this can is the same shape as this hoop; find parts of the room that are this shape).

TRANSFORMATION AND SYMMETRY

Experiences in transformation and symmetry should be provided which enable children to:

A5 recognise simple translations, rotations and reflections, and related symmetries

Possible activities:

- Fold objects (e.g. jumpers, paper shapes) to test for reflection symmetry and draw lines of symmetry in shapes (e.g. letters, drawings of cats).
- Use varied techniques (e.g. 'inkblot' paintings, potato prints, fold and cut paper, rotate pattern blocks) to make symmetrical patterns (rotational, reflectional or translational).
- Classify patterns into those based on rotations, reflections and translations, respectively, and investigate their use in papers and fabrics.
- Investigate transformation and reflection symmetries in natural things, in the art of various cultures, playground equipment and familiar toys, and discuss any purposes they serve.
- Sort objects into those which rotate through a full turn (e.g. helicopter rotor, clock hands) and those which rotate through part of a full turn (e.g. swings, scissors).
- Observe and investigate transformations which distort shape and size (changes in the shape of a figure when the balloon on which it is drawn is blown up; reflections on the back of a spoon).

A6 rearrange, fit together and tessellate figures, and stack and pack objects

Possible activities:

- Complete games and puzzles which require fitting pieces together in specified ways (e.g. jigsaw puzzles, tangram puzzles).
- Experiment with simple figures to see whether they will or will not fit together, discussing features of figures (i.e. corners, sides) which relate to whether or not they tessellate, and hypothesising about which shapes do and do not tessellate (e.g. all the triangles I have tried tessellate; I think they all do).
- Identify and make patterns based on simple tessellating shapes.
- Experiment with simple objects (e.g. drink cans, blocks) to see whether they will or will not stack or pack, discussing features of the objects which make them stack or pack.
- Cover the surfaces of a range of shapes and discuss the most appropriate way.

A7 make informal use of scale in producing models and drawings

Possible activities:

- Play with toys and models, discussing size and scale informally (e.g. this teddy looks about right to go with this chair, but that teddy is far too big).
- Make models to a specified but informal scale (e.g. use plasticine to make a table about the right size for the house).
- Arrange objects in sequence and match series (e.g. given clowns of different sizes, match with shoes).
- Make a shape on square paper or with cubes, and make a different size copy.
- Make plans and models of familiar structures using informal scale.

- Consider the location and relative size of drawn objects in drawing and photographs (e.g. the drawing of this child is bigger than the drawing of her father: why?).

A8 recognise realistic two-dimensional representations of objects

Possible activities:

- Post blocks through holes.
- Compare photographs and realistic drawings (e.g. of self, friends, and objects) with the people or things, discussing what can and cannot be seen.
- Recognise realistic drawings and photographs of familiar objects (e.g. ice-cream, chair) and geometric models (e.g. cone, prism) from different viewpoints and match the object.
- Draw objects from different viewpoints.

LOCATION AND ARRANGEMENT

Experiences in location and arrangement should be provided which enable children to.

A9 give and follow directions for moving and locating objects

Possible activities:

- Follow (e.g. act out a story) and give directions (e.g. the best way to go to the tuckshop from here) using language of location and movement.
- Make informal maps and refine through use with other children.
- Participate in movement games and dances of turning, direction and location.
- Give and follow simple directions, involving quarter, half and three-quarter turns, and right angles, to locate objects in models, plans, maps or mazes, or on a computer screen.
- Use informal grids to locate objects (e.g. the book is on the third row, second from the end).
- Investigate routes solving simple problems (e.g. where should the paths be placed?).

A10 plan, describe and use arrangements in practical settings

Possible activities:

- Plan and arrange toys and models (e.g. farm set, a puppet stage setting).
- Compare different arrangements (e.g. of rooms, gardens, playgrounds) for particular purposes.
- Plan and re-arrange local environments according to specifications (e.g. all the vegetables will get sun and water and we will be able to reach them without walking over other plants).
- Use the language of inside, outside, on, boundary, open and closed appropriately in context.

SPACE in BAND B

Overview

During band B, the approach should continue to be informal and practical and emphasise the investigation of the features of objects in the environment, including their shape and the effect on them of changes in shape, size or position.

Children should be able to think of classes of shapes and analyse the properties that distinguish members of one class from members of a different class. For example, they may say 'These shapes all belong together because they all have at least one square face. But this shape [the square pyramid] is different from the rest; it has triangular faces and they don't'.

Children should 'tinker' to find out how mechanisms function and build increasingly complex structures and mechanisms. They should predict the effects of changes they make to the shape, size or position of figures and objects and check by experimenting. For example, they may say 'I decided that if I moved this box along the desk in a straight line, all its corners would move the same distance. I did this and measured and found out I was right'.

There should be plenty of opportunity for children to discuss their findings and to practise ways of describing space mathematically. Over time, they should refine their descriptions of what they see and do. Without using unduly technical vocabulary or formal language styles, teachers can use correct spatial language repeatedly in contexts which make the meanings clear. Students should use the language for themselves (rather than simply following it when used by others) and speak with increasing clarity and precision, but the testing of vocabulary as such is not expected.

Geometric drawing equipment such as compasses and Miras™ should be used during these years and children should develop their skill and precision in drawing shapes. The ability to interpret and produce representations of three-dimensional shapes develops slowly and children will need considerable experience with these. This may be integrated with art and craft work.

Making reduced or enlarged copies of pictures or objects, perhaps in art or social studies, can provide an excellent context for the development of fractional and ratio concepts (and, later, a meaning for multiplication by fractions). This leads eventually into the more technical idea of scale. During band B, there should be a gradual development from a very intuitive feeling for scale to the somewhat more formal use with whole numbers or unit fractions as scale factors. The relationship between number, measurement and space is particularly visible in the investigation of scale and the effect of change of scale on such measures as area and volume.

Children's repertoires of ways to represent and describe location should include simple grids and distances and directions. They should relate direction and angle of turning to compass directions and use a magnetic compass to determine simple directions. In addition, they should develop intuitive ideas about pathways and networks.

SHAPE AND STRUCTURE

Experiences in shape and structure should be provided which enable children to:

B1 build structures and make geometric models, analysing their cross-sections and nets

Possible activities:

- Make models of familiar structures (such as a table) using a range of construction materials.
- Investigate similarities and differences in a selection of shelters used historically and presently, considering possible reasons for the use of the shape and construction methods.
- Make models of spheres, cylinders, cones, prisms and pyramids (using plasticine, clay, play dough, food), analysing and discussing the attributes of each object.
- Predict the shape of sections of environmental objects (e.g. carrots) and geometric models (e.g. cones) and slice models to test predictions.
- Make packages (such as juice boxes) from nets which are provided or made by using templates or tracing around the object itself.
- Predict nets for particular boxes; cut identical boxes in different ways to investigate alternative nets for the same object.
- Design and make packages as specified (e.g. send an odd-shaped Lego construction through the mail). Some such problems can also be investigated using computer packages.

B2 compare and classify objects, analyse their shapes and describe in conventional geometric language

Possible activities:

- Compare and classify objects according to geometric features (e.g. all of these stack, but these don't), explaining the criteria for sorting.
- Describe likenesses and differences between objects using terms such as faces, edges, parallel faces, angles, cylinder, cone and pyramid.
- Identify and name parallel, perpendicular, horizontal and vertical lines and surfaces in the environment and on models.
- Investigate and report on strength, durability, functionality, rigidity or flexibility and aesthetic appeal of structures made by students.
- Compare natural shapes (e.g. leaves of plants) from different environments, considering the adaptations of shape to function.

B3 make and draw a variety of plane figures using geometric tools

Possible activities:

- From a verbal description of a plane shape, draw it on dot paper or produce it on a geoboard (e.g. can you make a triangle on your geoboard with all sides equal? … two sides equal?).
- Make a variety of figures on a geoboard and copy and classify them.
- Use a ruler and set square to draw parallel and perpendicular lines and use a ruler, protractor, set square and compass to construct circles, triangles, squares and rectangles.
- Use a variety of techniques (e.g. template, compass, string and pin) for drawing

circles according to specification (e.g. location of centre, radius, diameter).

- Solve construction problems (e.g. draw the biggest triangle which will fit inside the circle).
- Investigate and design logos (e.g. the Aboriginal flag or a flag of one's own design).
- Use a computer graphics package to design 2-D shapes such as a stylised picture of a house or a pattern composed of simple shapes.

B4 compare and classify figures, analyse their shapes and describe in conventional geometric language

Possible activities:

- Compare and classify plane figures according to geometric features (e.g. these are easy to tile and those aren't), explaining the criteria for sorting.
- Compare and classify plane figures according to criteria suggested by others, using geometric language as needed (e.g. side, angle, circle, diagonal, section, parallel, vertical).
- Compare and classify various types of quadrilaterals and triangles and investigate to find relationships (e.g. what shapes can be made by placing two identical triangles edge to edge? what if they are right triangles? … right isosceles triangles?).
- Compare and classify polygons (triangle, quadrilateral, pentagon, hexagon and octagon) and investigate to find simple relationships (e.g. the more sides in a regular polygon the greater the interior angles).

TRANSFORMATION AND SYMMETRY

Experiences in transformation and symmetry should be provided which enable children to:

B5 carry out rotations, translations and reflections, and recognise and produce related symmetries

Possible activities:

- Make a shape from cubes. Construct another, given a specified transformation or rotation (e.g. build what you think you would see after a quarter turn, ... in the mirror).
- Given a figure or object and a simple verbal description of a transformation, draw or make the resulting shape.
- Use prints (stamps, potato prints) and stencils (templates, paper cutouts) to make (rotational, reflection or translational) symmetry patterns.
- Identify, describe and continue spatial patterns based on symmetry, simple movements and sequences, such as ..˙..˙.˙.
- Identify transformations and symmetries in fabrics or wrapping papers, trademarks and logos, music, drama and dance.
- Construct kites and origami objects, investigating the practical and aesthetic purposes of their symmetries.
- Investigate changes in orientation in reflected objects (e.g. is the left shoe a reflection of the right shoe? if you lift your left arm, which arm does your reflection seem to lift?).

B6 recognise congruent figures by superimposition, and informally investigate congruent figures

Possible activities:

- Superimpose figures to check congruence of angles and lengths, and compare lengths and angles of pairs of figures to check for congruence.
- Fold and cut paper figures in half in various ways; check whether the halves are congruent and describe the movement needed to superimpose one half on the other.
- Fold and cut plane shapes in various ways to form a given number of congruent parts.
- Identify congruent faces on polyhedra and use to identify prisms, pyramids and shapes that are neither, such as dodecahedrons.

B7 rearrange, fit together and tessellate figures, and stack and pack objects

Possible activities:

- Reassemble puzzle pieces as for jigsaws, tangrams or mosaics.
- Make tessellations of shapes, and describe how pieces fit together, using the language of slide, flip and turn (or translation, reflection and rotation).
- Make modifications to tessellation shapes to make designs (e.g. like those of Maurits Escher).
- Predict and test the packing and stacking properties of common objects (e.g. bricks, milk cartons and bottles, marbles) and solve stacking and fitting puzzles (e.g. Soma cube).
- Investigate various ways of laying bricks for strength, durability, rigidity or flexibility and aesthetic appeal.
- Research the origins of some traditional and modern puzzles involving the rearrangement of parts (e.g. jigsaw, tangram puzzles, Soma cube, Rubik's cube).

B8 reduce and enlarge figures and objects and investigate distortions resulting from transformations

Possible activities:

- Use blocks to create objects, or draw figures on grid paper, and enlarge or reduce to specified scales.
- Overlay a grid on existing figures (e.g. a map of Australia) and enlarge or reduce by using a different size grid.
- Draw shapes on grid paper and produce distortions by changing either horizontal and vertical scales but not both (e.g. double heights).
- Use projective light sources to make shadows which are enlargements of shapes, draw around shadows to make a record and check the shape.
- Distinguish conditions under which a light source produces shadows which are enlargements or distortions of the original shape.
- Investigate changes in areas and volumes as a result of enlargements and reductions (relate to **Measurement** and **Choosing and using mathematics**).

B9 produce, interpret and compare scale drawings and maps, and scale models of familiar structures

Possible activities:

- Make scale models and drawings of familiar structures and places, using simple scale factors (whole numbers or unit fractions).
- Interpret sketches of localities (e.g. explain what has changed and what has remained the same in the various sketches of the same locality over a period of time).
- Interpret the use of scale on maps (e.g. map of Australia), plans (house plan) and diagrams (model-making instructions).
- Compare the scale on two different maps or plans and explain reasons for using the same scale on component parts of a scale drawing or model.
- Investigate the scale of toys and models considering the relationships between the parts.
- Given the scale of a model (e.g. wooden dinosaur skeleton), calculate the dimensions of the original.

B10 interpret and produce two-dimensional representations of objects

Possible activities:

- Draw environmental objects and geometric models from different viewpoints (side, top, front).
- Identify common environmental objects and geometric shapes (e.g. spheres, cylinders, cones, prisms) in drawings, paintings and photographs taken from different viewpoints.
- Discuss what is and is not seen from a particular view of a prism or pyramid and sketch it.
- Given a drawing or photograph of a 3-D object, draw it from a different viewpoint.
- From side, top and/or front views of models made with cubes, predict how many cubes are needed to build the object and then build it.
- Understand and use the convention for drawing objects — visible edges drawn with solid lines and hidden edges with dotted lines.
- Place a collection of photographs (e.g. of a building, still-life) taken from different orientations in order as you move around.

LOCATION AND ARRANGEMENT

Experiences in location and arrangement should be provided which enable children to:

B11 give and follow directions for moving and locating objects, using angles, compass points and grids

Possible activities:

- Follow and give directions involving turns (e.g. walk 5 m towards the the tree, take a quarter turn to the left and walk …) and discuss ambiguities in directions in order to improve them.
- Relate angles as 'amounts of turning' to dance, sport and games (e.g. ballet, skateboard turns) and familiar technology (scissors, steering wheel).
- Using a computer graphics package or programming language (e.g. Logo) devise instructions to follow a path or produce a figure.
- Use 360° as the angle measure of a complete turn to calculate or estimate the size of smaller angles (e.g. a quarter turn must be 90°) and relate degrees and direction of turning to compass directions.
- Give clear sets of directions orally and interpret oral directions with a mental map (e.g. children sit back-to-back; one gives a set of directions which the other tries to sketch).
- Draw routes on a grid (e.g. Find the shortest route a taxi could take to get from this corner to that corner. Are there other equally short routes?).

B12 plan and execute arrangements according to specifications

Possible activities:

- Investigate the arrangement and layout of familiar structures (e.g. the school, the library, kitchen and house plans) for efficiency of use and movement between regions.
- Consider the practical importance of the orientation and location of buildings by researching, for example, the placement of houses and gardens for climate control.
- Consider the spiritual importance people have placed on orientation and location by researching buildings and structures (e.g. early Christian churches faced east or the direction of the sunrise on the day of the saint for which the church was named; Muslim mosques face east).

B13 describe, follow and record paths and routes using sketches and simple networks

Possible activities:

- Draw simple networks of familiar places (e.g. home, library) showing key regions and paths.
- Investigate routes between locations (e.g. in how many ways could you go from school to your home? can you get the knight back to the starting point? in how few moves?) and draw paths and tracks to show the use of roads and paths.
- Informally investigate flight paths in familiar situations, such as the flight of paper planes and various balls (netball, football, tennis ball).
- Given descriptions of flight paths, draw sketches to represent them (e.g. the flight of Jonathan Livingston Seagull).
- Identify and record routes on networks which represent familiar locations or objects (e.g. from school to home, or a knight on a chess board).

SPACE in BAND C

Overview

As indicated for bands A and B, students will vary in the spatial experiences they bring to the mathematics classroom and in the language with which they are comfortably familiar. Learning experiences and assessment tasks in geometry should draw equally on the experiences of students from different backgrounds and of girls and boys, but should extend their skills and the contexts to which they can apply geometric knowledge.

All students in band C should:

- examine two-dimensional and three-dimensional geometric shapes, and formulate relationships between classes of shapes (e.g. squares are rhombuses and all squares are also rectangles, but not all rhombuses and rectangles are squares);
- visualise and describe the effect of reflections, rotations, translations and enlargements on shape, size, orientation and arrangement, and recognise symmetries;
- recognise, analyse and describe distortions of shape and size;
- improve their proficiency at drawing figures and constructing objects, and their use of the conventions for representing three-dimensional objects in two dimensions;
- apply this knowledge to problems.

Not all of the scope statements should be treated in the same depth. For example, while students should experience both transformational and Euclidean approaches to the investigation of space, a particular curriculum may emphasise one more than the other. Rather than treating both superficially, one may be chosen for indepth and possibly more formal treatment, while the other is considered informally and intuitively. If transformations were emphasised, then one would expect this to go beyond the analysis of particular transformations to the formulation of relationships between them. Thus students may say, 'If you have two parallel reflection lines and carry out two reflections over them, it is as though you had translated the shape. Also the distance you have translated it is twice the distance between the reflection lines.'

Similarly, within *Location and arrangement*, networks and loci have been suggested as two different ways of considering routes, paths and regions. It may be useful to compare these approaches and consider their historical and present significance, but it is not anticipated that each would receive the same emphasis.

The importance of proof in mathematics is described under *Justification* in the **Mathematical inquiry** strand. In the past, traditional Euclidean geometry has been regarded as the major vehicle for the development of deductive proof in school. Such approaches have not been successful for the majority of students. Notions of proof need to be developed in all the strands, however, and geometry can contribute in a range of ways.

- Confronted with a practical problem (e.g. to ensure that bases for a softball game are always correctly placed), students may use their geometric knowledge to develop and justify a method.

- Students may design Mira™ construction techniques and, taking the properties of reflections as axioms, prove the constructions will always work.
- Using one form of the parallel postulate as an axiom, students may decide whether or not pairs of lines are parallel and develop arguments to justify their conclusions.
- On the basis of a set of properties of a shape (or a diagram and given information) students work in groups to generate other properties by a process of conjecture and justification.

The emphasis should not be on remembering particular proofs but on the processes of proving. The process of conjecture and justification in geometry may be facilitated considerably by the use of software such as the Geometric Supposer™.

SHAPE AND STRUCTURE

Experiences in shape and structure should be provided which enable students to:

C1 construct objects satisfying design specifications

Possible activities:

- Design nets and construct objects (e.g. produce a model of the library building or a local store).
- Construct everyday objects which require careful folding of a net (e.g. milk carton) or 'knock down' object (e.g. magazine box), investigating the design features which make the objects 'work'.
- Construct cylinders and cones to specifications (e.g. construct a cylinder with a height of 15 cm and a diameter of 6 cm).
- Produce packages according to specifications (e.g. a box with a lid that slides over the top).
- Build skeleton models of 3-D objects to specifications (e.g. a polyhedron with 20 edges and 12 vertices; the school resources centre).

C2 recognise and apply features, sections and nets of polyhedra, spheres, cylinders and cones

Possible activities:

- Articulate the properties which make 3-D shapes members of classes (e.g. these all belong together because they all have only triangular faces) and formulate relationships between classes of shapes (e.g. all cubes are also prisms because ...).
- Visualise and match polyhedra with their nets (e.g. which pentominoes can be folded to make an open box?) and identify the shapes when shown in different orientations and embedded in more complex shapes.
- Visualise and predict sections of 3-D objects, including the conic sections, and distinguish shapes which do and do not have uniform cross-section.
- Investigate features of objects which make structures flexible or rigid, fragile or strong, efficient for packing or stacking, and cost-effective.
- Investigate social and economic issues related to packaging (e.g. will people buy products inexpensively packaged?).

C3 draw figures satisfying given conditions, using a variety of geometric techniques and tools

Possible activities:

- Make and investigate angles and angle bisectors by folding paper.
- Use a protractor, Proliner™, Geoliner™ or Rotagram™ to measure and produce angles (e.g. copy a design from a book).
- Use set square, Math-o-matt™ or Mathaid™ to draw right angles, parallel and perpendicular lines and squares and rectangles of given dimensions (e.g. produce a plan of the school).
- Investigate constructions which are possible using only a compass and ruler (e.g. copy angles and intervals, and construct circles, parallels, perpendiculars, right angles, bisectors of angles and intervals, triangles and quadrilaterals).
- Investigate constructions which are possible using only a Mira™ and ruler (e.g. copy angles and intervals, construct parallels, bisect angles and intervals, construct quadrilaterals).

- Represent situations in diagrams (e.g. as they occur in measurement contexts), including those with insufficient information.

C4 use properties of angles, intersecting and parallel lines

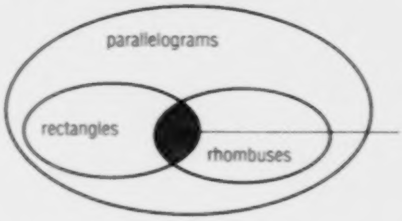
Possible activities:

- Develop and carry out practical checks for parallelism, perpendicularity, verticality and horizontality (e.g. use the corner of a sheet of paper to test for perpendicularity; a ball or spirit level to check if a surface is horizontal).
- Through a process of conjecture and justification, establish basic angle properties of parallel lines.
- Apply the basic angle properties of parallel lines (e.g. find a marked angle on a diagram, determine whether two lines are parallel, orienteering).
- Use computer graphics to investigate spatial situations involving angles and lines in the plane (e.g. the movement of a billiard ball around various-size tables).
- Construct a concept map to summarise the relationship between the concepts of line, angle, plane, parallel, perpendicular.

C5 formulate relationships between various polygons and use properties to solve problems

Possible activities:

- Articulate the properties which make polygons members of classes (e.g. these quadrilaterials are all rectangles because their diagonals bisect each other) and formulate relationships between classes of shapes (e.g. squares are the regular quadrilaterals, they ...).
- Use concept maps or Venn diagrams to demonstrate relationships between classes of shapes.



The shapes which are rhombuses and also rectangles are squares

- Use paper folding and tracing techniques to investigate geometric properties (e.g. angles, parallelism) of plane shapes.
- Investigate angle properties of polygons and the Pythagoras' theorem for triangles and apply to measurement problems (e.g. can a 2 m surf board fit inside a 0.8 m by 0.4 m by 1.9 m cupboard?).
- Through a process of conjecture and justification, establish triangle and quadrilateral properties (e.g. in any isosceles triangle two of the angles are equal and the sides opposite them are equal; the three angle bisectors intersect at the same point) and apply generalisations in everyday situations.
- Apply the properties of quadrilaterals in practical contexts (e.g. use the fact that the diagonals of a rectangle are the same length to ensure that the corners on a rectangular patch are square).

C6 formulate properties of circles and use to solve problems

Possible activities:

- Use the language associated with circles (e.g. centre, radius, diameter, chord, arc, semi-circle, circumference, segment, sector and tangent).
- Construct circles and ellipses (e.g. mark out circles for a netball court with rope of fixed length, connected to a focus) and find the centre of existing circles (e.g. to place a hole in the table).
- Investigate and apply simple chord, angle and tangent properties (e.g. each tangent is perpendicular to the diameter/radius at the point of intersection of the tangent with the circle).

TRANSFORMATION AND SYMMETRY

Experiences in transformation and symmetry should be provided which enable students to:

C7 use translations, rotations and reflections and the relationships between them

Possible activities:

- Experiment to find that length, angle size, area and volume do not change under translation, rotation and reflection.
- Identify a transformation which maps one figure onto another (e.g. the blade was rotated a quarter turn anticlockwise about the centre of the fan), and apply given transformations to a figure.
- Solve practical problems involving transformations (e.g. a new single bed has been bought for your bedroom; plan and describe the movements needed to get it into your room from the front door).
- Investigate rotation as used in simple machines and the means by which rotary motion is converted to linear motion and vice versa (e.g. spinning wheel, windmill, cams).
- Find and explain a general method for producing a translation (or rotation) using successive reflections.
- Express general arguments about properties of figures based on symmetry (e.g. if you cut a tessellating figure along a line of symmetry the half figure will also tessellate because you can ...).
- Investigate and explain why a particular given figure cannot tessellate.
- Identify symmetries in plane and solid shapes (e.g. use mirrors to investigate regular polygons and relate to kaleidoscopes) and record symmetries in polyhedra (e.g. a prism).
- Research and report on some of the uses of transformation and symmetry in art (e.g. heraldic designs, the work of Escher, surrealism).

C8 identify congruent figures and formulate and apply congruence conditions

Possible activities:

- Experiment (using superimposition and measurement) to suggest sets of minimum conditions under which triangles will be congruent.
- Using congruence conditions as axioms, establish the congruence of two triangles and solve numerical problems (e.g. spacing trees in a plantation).
- Experiment to suggest sets of minimum conditions under which pairs of polygons (e.g. quadrilaterals) will be congruent.
- Use the properties of translations, reflection and rotation as axioms in order to establish the congruence of two shapes.

C9 enlarge and reduce figures and objects and formulate and apply similarity conditions

Possible activities:

- Enlarge or reduce a figure using a grid, pantograph, joined rubber bands or projection technique and enlarge objects by building with blocks or scaling the object's net.
- Investigate the effects of scaling plane and solid shapes considering such features as lengths, areas, volumes, angle measures, shape, parallelism, perpendicularity, ratio of mass to surface area.

- Estimate the scale factor relating an image to its original situation, using judgement and rough measures (e.g. given a photograph of oneself and a friend together, estimate the friend's height).
- Determine the scale factor relating an image to the original situation by measurement of corresponding lengths and calculation.
- Use similarity conditions for triangles to solve simple geometric problems (e.g. proving two triangles similar) and practical problems (e.g. heights from lengths of shadows).

C10 interpret and produce two-dimensional representations of objects

Possible activities:

- Given scale plans, drawings and models, and the scale factor, estimate actual sizes in the original situation.
- Construct maps, plans and models to scale from measurements planned and made by students (e.g. a plan of part of the local environment) or reference materials (e.g. a model of Parliament House).
- Make sketches of objects, choosing from amongst plans and elevations, isometric and oblique projections and one- and two-point perspective.
- Interpret everyday instances of 2-D representations of 3-D objects such as computer images, weather and contour maps, clothing patterns and exploded drawings for kit assembly.
- Sketch a shape on the basis of a description (e.g. the region which can be illuminated by a searchlight).
- Compare conventions in different eras and cultures for representing depth of field (e.g. compare a traditional Chinese landscape painting with an Australian landscape painting), considering the conventions of representation and the use of perspective.

LOCATION AND ARRANGEMENT

Experiences in location and arrangement should be provided which enable students to:

C11 give and follow directions for moving and locating objects, using angles, compass points and grids

Possible activities:

- Give clear and unambiguous instructions for other students to represent in a diagram (e.g. describe a camp site so that another student can draw a plan).
- Through games and puzzles represent ordered pairs and triples on two- and three-dimensional grid (e.g. treasure hunts, knight's moves on a chessboard, battleships, three-dimensional noughts and crosses).
- Specify location by means of coordinates (e.g. investigate computer screen systems used to label screen points).
- Compare coordinate systems (e.g. overlay cartesian or polar grids over maps or aerial photographs of regions and investigate uses, advantages and disadvantages of each).
- Use bearings to define direction.
- Work collaboratively to develop and justify a method for finding the distance between two points, given their coordinates.
- Work collaboratively to develop and justify a method for finding the point half-way between two points, given their coordinates.
- Use coordinate systems on a sphere, especially lines of latitude and longitude (e.g. use a globe to estimate the coordinates of Townsville in Queensland) and informally investigate circles and great circles (e.g. use a globe and rubber bands to investigate airline routes).

C12 describe, follow and record regions, paths and routes using networks and loci

Possible activities:

- Find shortest routes and paths between locations (e.g. a mail route).
- Distinguish networks from maps or plans.
- Draw and interpret simple networks to represent systems involved in transport, the provision of water, fuel and telephone, and body systems.
- Decide whether two networks are equivalent and whether particular networks are traversible.
- Visualise, describe and draw simple loci in 2-D and solve simple 2-D problems involving loci (e.g. design a watering system).
- Visualise, describe and draw solids generated by the rotation of lines and curves of different shapes (e.g. as in a potter's wheel or lathe) and recognise such solids of revolution in everyday life (lampshade, parabolic reflector in headlights).
- Investigate and describe the loci of different points on simple machine linkages.
- Use a variety of strategies for constructing ellipses, parabolas, hyperbolas and spirals.

SPACE in BAND D

Overview

During band D, all students should be expected to apply geometric concepts and results to practical problems. Using computer packages they may also formulate and test hypotheses in three dimensions and apply what they have learned to complex design tasks. They should examine a range of applications of geometry in fields other than mathematics (e.g. scaling is used in the biological sciences to identify limiting factors on the growth of various organisms) even if at times the technical demands of the applications go beyond what students are expected to produce independently.

Many applications of mathematics depend upon the capacity to visualise and represent objects and transformations of those objects. The development of these skills should be explicitly addressed. Students will continue to need practical experiences with two- and three-dimensional shapes. 'Thought experiments' in which they imagine, for example, the effect of two spheres coming across each other's paths and moving through each other, can be followed up by the inspection of computer-generated images of these dynamic processes.

Students should have experiences with a variety of analytical techniques for describing two- and three-dimensional space (e.g. coordinate, vector, Euclidean, transformational, networks). Their understandings of congruence and of similarity and scale, for example, can be deepened considerably by linking the various methods of geometric analysis. They should compare the strengths and limitations of various analytic techniques in specific practical situations. Each of these analytic techniques has its own vocabulary, notation and style of argument, and students should learn to use these forms with precision and clarity.

Students should also begin to relate various fields of mathematics to each other. They should, for example, understand coordinate geometry as the synthesis of algebraic and geometric techniques and recognise transformations as functions. These understandings are generally not well developed unless the links are made explicit and assistance is provided for students to bring a variety of geometric and other methods of analysis to bear on the same problem situations.

The **Space** strand provides an opportunity to compare and contrast axiomatic systems. A formal approach to axiomatic methods is likely to be inappropriate for most students. However, the general ideas are of such historical and current significance in understanding mathematics that some attention to them may be appropriate. For example, students may be introduced informally, perhaps from a historical perspective, to the notion of non-Euclidean geometries. In this case, they should recognise that Euclidean and non-Euclidean geometries are not simply different analytic tools; rather they provide contradictory models of the universe. Some students might also link this to Newtonian and Einsteinian physics.

Given the range of new content in school mathematics curricula, it is no longer realistic to expect even mathematically very able students to develop Euclidean geometry from axioms. For some students, however, it may be appropriate to develop skills in reasoning deductively from assumptions within the context of specific areas of

Euclidean geometry (e.g. circle theorems).

Students should recognise that networks can be used in situations where measurements such as length or angle can be ignored. Hence, they may be used when we cannot or do not need to make assumptions about such matters as straightness, parallelism, congruence or similarity. Many problems related to optimisation of job allocation, traffic flow and location are accessible on the basis of very simple network techniques although, where many calculations are required, the use of a computer is essential.

SPACE

Experiences in space should be provided which enable students to:

D1 visualise and represent in two- and three-dimensional space

Possible activities:

- Construct objects using a variety of materials given certain information (e.g. given drinking straws and string, construct an object having 6 equal sides, or comprising 8 equilateral triangles).
- Identify various geometric features in a model (e.g. given a polyhedra model, find examples of 2 parallel lines which determine a plane).
- Investigate how conic sections can be generated using various methods (e.g. cut a cone by a plane, cut a right cylinder by a plane, project a sphere, ...).
- Use CAD software to explore designs and geometric properties of objects in three dimensions.
- Use formal methods to represent three-dimensional geometrical shapes (e.g. coordinates, vectors, ...).
- Produce network diagrams of polyhedra and find Eulerian and Hamiltonian paths.
- Research the origins of different geographic map projections, the mathematics underlying them and the distortions inherent in them.

D2 use the language and techniques of coordinate geometry

Possible activities:

- Research the history of coordinate geometry.
- Use the language and techniques of coordinate geometry to describe and derive properties of two- and three-dimensional figures.
- Use algebraic representations of simple plane shapes (e.g. circle, parabola, ellipse).
- Use methods of coordinate geometry to describe lines fitted to data scatter plots.
- Apply methods of coordinate geometry to the solution of practical problems (e.g. obtaining the intersection of two lines which are modelling trends in finance).
- Verify results using coordinate methods (e.g. cosine rule, Euclidean results).

D3 solve problems, choosing analytical tools from amongst Euclidean, transformation, vector and coordinate geometries, networks and trigonometry

Possible activities:

- Use the geometrical properties of figures to solve problems involving chance (e.g. what is the probability of obtaining a bull's eye on a particular target, given that the target is hit?).
- Use geometric knowledge in maximisation problems (e.g. given two sides of a triangle, maximise the area).
- Use and discuss geometrical properties in relation to measurement tasks (such as mapping, surveying, orienteering, navigation).
- Use geometric properties to solve design problems (e.g. use tessellation in the plane, the relationship between shape and function,...) using drawing instruments and CAD packages.
- Compare ways of describing movement of design motifs on a computer screen (e.g. using the language of transformations, vectors, coordinates).

- Carry out simple deductive proofs in Euclidean geometry and/or equivalent proofs using vector or coordinate means.
- Recognise the same notion in different analytic forms (e.g. scalar multiplication of vectors, the enlargement transformation, similarity of plane figures in Euclidean geometry seen as change of linear dimension).

D4 recognise that geometries are invented systems

Possible activities:

- Research the history and applications of Euclidean and non-Euclidean geometries.
- Discuss the contradictory nature of various internally consistent models of space (Euclidean, Riemannian, Lobachevskian).
- Use networks to represent physical and social systems (e.g. sequences of job allocations, possible delivery routes for a vendor) and recognise that these systems indicate properties such as 'betweenness', and number of nodes, but do not require measures.
- Identify the properties of various geometries (e.g. meaning of parallelism, effect of mappings).
- Use and appraise different ways of modelling space (e.g. various map projections).

Number

Purpose and scope

All people need to develop a good sense of number, that is, ease and familiarity with and intuition about numbers. This requires a sound grasp of number concepts and notation, familiarity with number patterns and relationships, a working repertoire of number skills and, most importantly, confidence in one's capacity to deal with numerical situations.

'Playing' with numbers is important in the development of confidence and competence in their use. This may mean comparing strategies for a mental calculation, or pattern exploration, or investigating questions like 'What if our place value system used eights instead of tens — what would change?' or 'What if sales tax was added after a discount was taken rather than before?'. This kind of exploration has quite utilitarian purposes. For example, spatial and number patterns may be used in mental computation or to break a code. Exploration can also help students develop control over their own learning processes and can allow them to share some of the fascination with numbers that can accompany such control.

Students should also learn about the development of number and computation in various cultures, coming to understand the way the *idea* of number pervades our society, and the significance of the development of number notations, and successively more efficient strategies for computation.

This strand is described under two subheadings: *Number and numeration* and *Computation and estimation*.

Number and numeration

Numbers are used to order and count and are applied to measurement. Part of the power of mathematics lies in the efficiency and flexibility of the many ways that numbers can be written. Everyday numeracy requires that one recognise:

- numbers in different forms (e.g. common fractions, decimal fractions, percentages);
- common equivalences (e.g. one-quarter is $^1/_4$, two-eighths, 0.25 and 25%);
- the order (e.g. 6.32 comes after 6.316) and relative magnitudes (e.g. '25% off' is not as good a discount as 'one-third off') of numbers in common forms.

Beyond simple counting, all calculations with whole numbers and decimal fractions make use of place value, and this concept should be developed and extended throughout schooling, beginning with counting numbers, generalising into large numbers (millions, billions) and decimal fractions. Numbers may also be expressed in 'standard form' (e.g. 37165.426 is written as 3.7165426×10^4) with a further adaptation for many scientific calculators (e.g. 3.7165426 $\underline{4}$). Students should experience the use of standard form for very small numbers (as used to measure, say, chromosome size) and large numbers (as for star distances).

As they proceed through the secondary years, students' understanding of real numbers should become more sophisticated. For some students, including those wishing to proceed to tertiary studies in mathematics or mathematically dependent fields, numbers should be extended to include complex numbers, and subsets of the real numbers should be investigated in more abstract terms.

Computation and estimation

All school leavers should feel confident in their capacity to deal with the computational situations which they meet daily, and number work should reflect the balance of number techniques in regular adult use. Firstly, in any given situation, people need to decide how accurate the answer must be. For example, a person who shops each week may make a rough *estimate* of the cost of a particular week's shopping list by making a judgement of how it compares with recent past weeks. Such an estimate may be used to decide how much cash to withdraw from the bank prior to shopping or to check the reasonableness of the charge made at the check-out counter after shopping. In such cases, no actual computation is carried out; rather a judgement is made from experience. Even when computations are carried out they may be either approximate or exact. A shopper with a limited amount of cash available may *approximate* the cost of a particular collection of items by rounding up and adding the rounded amounts; the same shopper would not be happy if the shopkeeper calculated the account the same way! People with number sense have learned to judge from the context how accurate they need to be. Whether for an exact or approximate computation, the necessary calculations are then carried out *mentally*, on a *calculator*, or using a *paper-and-pencil* method.

Students should develop the ability to judge the level of accuracy needed, learn to estimate and approximate, and use mental, calculator and paper-and-pencil strategies effectively and appropriately in different situations. For most students, adequate computational skill involves facility with the operations of addition, subtraction, multiplication and division with decimal numbers. This requires that they:

- decide what operations to perform (formulate the calculation);
- select a means of carrying out the operation (choose a method of calculation);
- perform the operation (carry out the calculation);
- make sense of the answer (interpret the results of the calculation).

Estimation and approximate computations

The distinction between estimation and approximation depends upon the degree of prediction involved. When we estimate we are using judgement based on hunch or experience in previous similar situations to predict a number or measure (e.g. It looks like there are about 350 people in a crowd). When we approximate we have a count or measure, but have chosen either to collect it to a chosen unit (e.g. information collected to the nearest dollar) or to round it (e.g. rounding the number of people who came through the turnstile to 340). The approximate number or measures may then be used in computation (e.g. The gate takings should be about 340 x $8). The distinction is subtle and many people choose to use the term 'estimation' for both. Where an estimate is arrived at from approximate values, students should assess the consequences for the accuracy of the estimate.

Good estimation and approximation skills enhance our ability to deal with everyday quantitative situations. They also assist in effective calculator use, providing a way of checking that a problem has been correctly formulated for the calculator and the keystrokes have been executed correctly. Approximate computations are based on single- or double-digit mental arithmetic, together with an understanding of place value. This reinforces the need for good place value concepts and skills in mental computation.

Mental computation

People need to carry out straightforward calculations mentally, and students should regard mental arithmetic as a first resort in many situations where a calculation is needed. Strategies associated with mental computation should be developed explicitly throughout the schooling years, and should not be restricted to the recall of basic facts. People who are competent in mental computation tend to use a range of personal methods which are adapted to suit the particular numbers and situation. Therefore, students should be encouraged to develop personal mental computation strategies, to experiment with and compare strategies used by others, and to choose from amongst their available strategies to suit their own strengths and the particular context.

Calculators and computers

All students should leave school knowing how to use a calculator effectively. It should be taken for granted that a calculator is available whenever it can be used, from years 1–12. A number of skills, both conceptual and technical, underlie the correct use of a calculator and these should be taught explicitly.

Computers are now the basic tool for computation in commerce and industry. Most consumers are affected by the use of computers as 'number crunchers' and many will work with them. As computers become available for ready use in school mathematics, students should have experience of their use for numerical computation using a variety of tools, including, for example, spreadsheets.

Probably the most common use of mathematics for the majority of people relates to money and finance. Concepts of money and exchange, leading into social and commercial arithmetic, should be regarded as important applications of the work on number and will provide fertile contexts for exploring the use of computers to investigate financial questions.

Paper-and-pencil computation

People use paper-and-pencil strategies as a memory device to extend the range of calculations beyond those easily performed mentally. We should expect paper-and-pencil techniques to continue to be in daily use for addition and subtraction and for multiplication and division by single-digit numbers. People with good number skills, however, usually do not use the standard algorithms taught in classrooms. Rather, they use a variety of informal but reliable written methods which are appropriate to the person, to the numbers involved (e.g. 27 x 19 and 27 x 17 may be performed differently) and to the context (e.g. on the telephone with paper handy or driving the car). Therefore, it is inappropriate to focus upon computation in isolation from the context of its use.

The development of flexible computational skills can be inhibited by emphasising the practice of standard paper-and-pencil methods to the exclusion of other methods. It is far more realistic to use a combination of mental and informal written methods most of the time, with paper-and-pencil recording seen as providing memory support. Calculators should be used for more laborious or repetitive computations. Access to this tool allows children to develop mathematical concepts more thoroughly and to concentrate on more realistic applications. Consequently, less emphasis should be given to standard paper-and-pencil algorithms and, to the extent that they continue to be taught, they should be taught at later stages in schooling.

Computational strategies such as those associated with permutations and combinations and sequences and series should also be developed during the later secondary years.

NUMBER in BAND A

Overview

Even amongst families in which counting and quantifying are a normal part of daily life there will be differences in the extent to which number concepts and skills have been explicitly used with, and encouraged in, young children. A wide range of experiences will be needed to ensure that *all* young children see the need to use numbers in counting, ordering and measuring. These include:

- representing counting numbers with physical materials, discussing and reflecting upon the effects of their actions so that mental representations of numbers are formed;
- using a calculator to investigate number sequences;
- interpreting and using ordering numbers (e.g. page numbers of a book), noting patterns (e.g. of the right-hand pages);
- reading and placing numbers on scales (e.g. clock, thermometer, ruler) and using these scales to measure.

Children should also recognise that sometimes numerals are used simply as labels, like letters, and do not imply quantity or order (e.g. postcodes, telephone numbers).

From the earliest years, number patterns should be seen as the basic building blocks for children's understanding of the structure of number. A feeling for the way numbers 'work' is a major aim and its achievement requires action and reflection by children in investigating patterns in number.

Young children should investigate realistic situations in which the need to add, subtract, multiply and divide numbers arises. These examples should enable them to see that one mathematical operation can apply to apparently quite different situations. For example, the division operation is appropriate for sharing problems (e.g. 18 cards, shared with 3 people, how many cards each?) and for grouping problems (e.g. 18 cards, 3 cards for each person, how many people?). The correct operation, and sequence of keystrokes, is 18 ÷ 3, in each case. The children should also begin to make explicit choices from amongst operations to apply in a given situation (see band A in **Choosing and using mathematics**).

The adoption of new conventions in terms of mathematical vocabulary, symbols or notation can cause considerable difficulty for young children, as it can for people of all ages. Such names and mathematical symbols may initially be difficult to discriminate. Consequently, children should express mathematical ideas in a variety of ways including talking, writing and drawing, gradually using conventional language and notation (see band A in **Mathematical inquiry**).

Using concrete materials, diagrams and calculators, they can investigate the basic sums and products (to 10 + 10 and 10 x 10), record their findings in a range of ways, eventually using conventional notation and making systematic records. While they may begin to commit some of these basic facts to memory, they are not expected to memorise them fully until band B.

Through practical problem-solving situations it should become clear to children that they need to operate on numbers larger than those that are accommodated by the basic facts. They may then use manipulative materials, calculators or other strategies to investigate methods for carrying out the computations which arise. Their mental computation should develop through discussion, comparison and reflection upon strategies and a great deal of varied practice.

The majority of mental and paper-and-pencil computational strategies are based upon the use of place value to write numbers, reinforcing the need for people to have a thorough understanding of place value and its relationship to computational strategies. While the ability to make full use of place value develops over many years, the foundations must be carefully laid during the early years through trading games, counting and grouping activities, and calculator explorations. Young children should be encouraged to explore and invent informal paper-and-pencil methods to supplement mental and calculator methods.

NUMBER AND NUMERATION

Experiences in number and numeration should be provided which enable children to:

A1 compare and match objects and collections

Possible activities:

- Match objects by one-to-one correspondence in meaningful contexts.
- Use one-to-one correspondence to compare collections of objects where the comparison serves a purpose which makes sense to the children.
- Use one-to-one correspondence to order collections (e.g. compare knuckle bone collections).
- Order objects by length (e.g. order children by height).
- Use the language of comparison (more, less, ...) in familiar situations across curriculum areas (e.g. reporting on growth of a plant, comparing family sizes, telling stories).

A2 use whole numbers to count and order

Possible activities:

- Make, count and order collections of objects (e.g. give me 6 pens; how many apples are there? put these groups in order from smallest to largest).
- Indicate the position of an object in a sequence (e.g. Maria is the fourth person).
- Read and write numerals and words for whole numbers, and say and order number names.
- Play counting games and sing counting songs from various cultures.
- Explore whole numbers which arise in measurement situations to develop a sense of relative 'size'.
- Devise strategies for counting collections (e.g. use groups of 2, 3, ...).
- Use unifix cubes to make 'staircases' in which each stair goes up by one block.
- Compare and order lengths made with unifix.
- Mark units along paper tape and use to find objects of specific length, that is, 2 units, 5 units, etc.
- Use a balance scale to compare masses (e.g. 3 units are heavier than 2 units).
- Use and compare different counting strategies in constructing basic facts (e.g. counting all, counting on, counting on from larger numbers, use double facts, make to ten).
- Identify numbers in everyday use and discuss which uses involve counting and which involve ordering.
- Identify numerals which are used as labels rather than to indicate counting or ordering (e.g. telephone numbers).

A3 recognise, produce and use patterns with whole numbers

Possible activities:

- Observe and describe regularities in number sets.
- Recognise, copy, continue and devise simple repeating patterns of numbers, and represent these patterns in spatial designs.
- Follow and create simple sets of instructions to generate number patterns and sequences.

- Explore and use the patterns in addition and subtraction relationships to 10, noting equivalent expressions for the same results.
- Explore patterns in number (e.g. in the 100 chart, using the repeated addition facility on a calculator).
- Identify number relationships (e.g. 4 nines is the same as 4 sixes plus 4 threes), by observing spatial representations.

A4 use place value concepts to read, write and compare whole numbers

Possible activities:

- Count and group collections and record the results (e.g. I had 11; I made three lots of 3 and two more, and then I made two lots of 5 and 1 …).
- Count collections and group in tens (I had 17; I found it was one lot of 10 and 7 over) in order to investigate the relationship with numerals.
- Play place value trading games with structured materials in order to develop the idea of the constant relationship between adjacent places and the skill of trading or exchange.
- Make, model and rename numbers in more than one way.
- Use a calculator or number strip to investigate the effect of constantly adding tens (e.g. 6, 16, 26, …) or multiples of ten (6, 26, 46, …).
- Use place value to read, write, say and order whole numbers of any practical size.
- Explore number names from languages relevant to the cultural backgrounds of the students and relate to place value.
- Use place value to round whole numbers for a practical purpose.

A5 recognise unit fractions (expressed in words and notation) as they occur in practical contexts

Possible activities:

- Interpret unit fractions which are expressed in words (e.g. give me half of the paper clips; give me half your sandwich).
- Describe simple equivalences of fractions which occur in everyday contexts (I painted a quarter of the fence before lunch, and then another quarter after lunch, so I have finished a half altogether).
- Investigate sharing in order to compare unit fractions (e.g. we shared the water — or beans — between three of us and that was a third each; then someone else came and we shared it between four — that was one-fourth each; one-third was more than one-fourth).
- Compare the size of unit fractions in practical situations (e.g. this recipe uses a third of a cup of sugar and the other one uses a quarter of a cup, so this one uses more).
- Read simple fraction notation in everyday contexts (e.g. correctly use fractions written in recipes).

A6 use two-digit decimals to express money and measurements

Possible activities :

- Use two-digit decimal fractions in the context of practical money activities (the book cost $2.95 and so I gave the shop assistant $3.00).
- Use two-digit decimal fractions in practical measurement activities.
- Read and enter on a calculator decimals which occur in practical situations involving money and measure.

COMPUTATION AND ESTIMATION

Experiences in computation and estimation should be provided which enable children to:

A7 make an appropriate choice of addition, subtraction, multiplication and division of whole numbers for a given context

Possible activities:

- Recognise practical situations where it is not possible to operate on the objects of concern and so it is necessary to operate on the numbers involved (e.g. I ate three sausages at lunch, and now I have eaten two more; how many have I eaten altogether?).
- Explore practical situations leading to the basic operations: addition; subtraction (as 'take away', difference or comparison); multiplication (as repeated addition); and division (as grouping and sharing).
- Investigate practical situations which can be represented by a particular operation (e.g. the 'subtract' key on a calculator is appropriate not only for 'take away' situations, but also for 'difference', 'count back' and 'comparison' situations.)
- Combine and split collections to investigate the relationship between addition and subtraction.
- Investigate relationships between multiplication and division.

A8 remember addition facts, and use mental arithmetic to extend these facts

Possible activities:

- Using concrete materials, diagrams and calculators, investigate basic addition facts and record results in a range of ways, including equivalent arithmetic statements (e.g. $4 + 5 = 9$ and also $4 = 9 - 5$ and $5 = 9 - 4$).
- Use and compare different counting strategies in constructing basic facts (e.g. counting all, counting on, counting on from larger numbers, use double facts, make to ten).
- Investigate patterns in a table of sums to $10 + 10$.
- Use concrete materials, diagrams and calculators to investigate the effect of multiplication by a multiple of 10.
- Use the fact that the order of the numbers is irrelevant when adding or multiplying two numbers.
- Use concrete materials, diagrams and calculators to show that order is relevant when subtracting or dividing (i.e. $36 \div 4 \neq 4 \div 36$).
- Use the fact that numbers can be added (multiplied) two at a time in any order.
- Engage in activities designed to help memorise the basic addition facts.
- Use place value patterns and relationships to assist with mental addition and subtraction of numbers bigger than 10 (e.g. $6 + 7 = 13$ so $16 + 7 = 23$, $26 + 7 = 33$).
- Devise mental strategies to extend basic facts (e.g. count in tens: 10, 20, 30, …; add and subtract in tens from any starting point: 16, 26, 36, 46, … ; add and subtract in multiples of ten from any starting point; count on and count back; multiply and divide by 10; double and halve).
- Give and check change by 'counting on'.
- Determine 'hidden' numbers in arithmetic situations (e.g. count 15 counters into a bag, take out a handful, count them and then say how many are left in the bag).

A9 use a basic four-function calculator efficiently

Possible activities:

- Enter and read measures and amounts of money and operate on them.
- Add and subtract any numbers arising in practical situations.
- Multiply and divide by whole numbers consistent with extent of concept development.
- Use constant multiplication and division techniques.

A10 estimate the size of a collection and approximate with whole numbers, measures and money

Possible activities:

- Estimate the size of collections of items (e.g. predict how many beans are in the jar).
- Find approximate numbers and amounts by rounding to whole numbers and, in the case of money, to the nearest dollar (e.g. if the pens cost $4.76 each, is $30 enough for 6?).
- Discuss with peers how accurate one needs to be and why, for particular situations.
- Investigate different ways of approximating (e.g. round up, round down) and discuss advantages and disadvantages in particular situations.
- Use a variety of strategies to approximate the result of additions and subtractions which occur in practical problem situations and make judgements about appropriate levels of accuracy.
- When using calculators, round to whole numbers and, in the case of money, to the nearest dollar.

A11 use efficient paper-and-pencil techniques to add and subtract whole numbers, measures and money, and use personal adaptations to multiply by single-digit numbers

Possible activities:

- Devise, compare and improve a range of personal written strategies to facilitate the accurate addition and subtraction of whole numbers.
- Practise efficient and reliable methods of adding and subtracting whole numbers and money.
- Devise and compare a variety of informal strategies to carry out multiplications which arise in problem situations.

A12 choose computational methods (mental, paper-and-pencil, calculator) and check the reasonableness of results

Possible activities:

- Choose appropriate means of completing calculations which occur in practical situations.
- Judge reasonableness of results, asking whether the input data are reasonable and the answer makes sense in context, and choosing and executing simple checks on accuracy.

NUMBER in BAND B

Overview

During these years, children should continue to have practical experiences which extend and generalise their number and place value concepts so that they can develop a sense of the large whole numbers (e.g. a sense of how big a million is) and small and large decimal numbers. They should also be encouraged to develop a variety of efficient counting strategies.

The investigation of patterns and relationships in number should continue to be a focus of work and children should identify, describe and continue patterns and create their own. The use of pattern should help children extend their understanding of large and small numbers and generalise their rules for operating on one- and two-digit numbers to numbers with more places. The work with patterns should also include multiples, factors, primes and composites, and some of the 'special' numbers, such as the figurate numbers (i.e. those that produce spatial patterns such as the triangular numbers or the square numbers). Their descriptions and representations of patterns and their ability to express generalisations should become more fluent during this period (see **Mathematical inquiry**).

The concept of decimal place value and the order and relative sizes of decimal numbers develop slowly. There is now considerable evidence that misconceptions about decimal fractions persist in many apparently successful students. For example, some will believe that in ordering decimals you can temporarily ignore the decimal point, order the whole numbers and then replace the decimal points. The problem with this particular miconception is that many children have learned to think about decimals in this way for computational purposes, in which case success in operating on decimals may actually reinforce the misconception! Providing a source of conflict about such interpretations and encouraging children to work together to resolve the conflict is likely to help them develop sound concepts of decimals. At the same time it should encourage them to believe that mathematics is supposed to make sense and that they can make sense of it.

The *meaning* of decimals and their relative order and size should be the focus of experiences in band B, and should relate to contexts such as ordering library books, interpreting scales, and comparing measurements.

Although decimalisation of money and measures reduces the need for computation with common fractions, fraction language and notation are in daily use and children should develop a good grasp of the common meanings attached to fractions. In band B, children will need to experience of the part/whole notion of common fractions with a wide range of discrete and continuous materials. While computational facility with fractions is not a priority for band B, the idea of, for example, adding fractions is an important part of concept of common fraction. Understanding how fractions are combined is essential in distinguishing the use of fractional notation to represent parts of a whole (e.g. $^1/_3$: I drank a third of the cordial) and using it to represent a comparison of one part with another part (e.g. $^1/_3$: the ratio of concentrate to water in the cordial was 1 to 3).

Children should be assisted to generate, read, use and appreciate different representations of numbers (common fractions, decimal fractions, percentages) or quantities in everyday contexts and begin to understand the historical and practical reasons for these various representations.

During band B, children will need to expand their concepts of the four basic operations. For example, they will need to expand their notion of the meaning of division if they are to use division in rate problems (see the comment on multiplication in band B of **Choosing and using mathematics**). By applying number skills to a wide range of practical situations, children should recognise the relevance of operations and computation to daily life and consolidate and extend the work of earlier years. They should devise a variety of paper-and-pencil computational strategies, record the results of their thinking in their own way, and progress towards efficient, although not necessarily standard, procedures. They should also compare their methods for efficiency and reliability.

Finally, estimation needs to be an ongoing part of children's study of numbers, and teaching should emphasise the development of a propensity to estimate. Children should begin to make decisions about whether a practical situation demands an exact or approximate solution and about how to calculate (mentally, calculator, paper-and-pencil) in a given situation. They should know what is meant by an estimate, when it is appropriate to estimate, and how close an estimate is required in a given situation. They should be helped to develop specific strategies to aid them in approximate computations.

NUMBER AND NUMERATION

Experiences in number and numeration should be provided which enable children to:

B1 use whole numbers and decimal fractions to count and order collections and measures

Possible activities:

- Read, write (in symbols and words) and say whole numbers of any size.
- Order decimal numbers in realistic situations such as replacing library books or comparing measurements from sporting events.
- Relate Dewey systems in libraries to decimal numbers.
- Order measurements in a meaningful context (e.g. here are the times for the swimmers in the race; who were first, second and third?).
- Devise strategies for counting large collections.
- Research and report on some of the different ways numbers are used as labels (e.g. postcodes and ISBNs).
- Construct a number line with units divided to one or two decimal places, and use the number to compare the lengths of various objects.
- Use a calculator to investigate patterns in decimal fractions (e.g. 0.2, 0.4, 0.6, …) and relate to the number line.

B2 recognise, produce and use patterns in number

Possible activities:

- Explain simple number sequences verbally and predict elements (e.g. 3, 6, 9, …, or 251, …, 273, …, 295, 306).
- Play one- and two-stage 'guess my rule' games and describe the rule verbally (e.g. whatever number I say, you add 1 and then multiply by 2).
- Make and explain an arithmetic generalisation (e.g. for all the products in the 9 times table, 9, 18, 27, …, 81, the sum of the digits is nine; I think this happens because …)
- Devise general rules to help with mental arithmetic (e.g. 7 add 6 is 13, so 17 add 6 is 10 add 7 add 6 which is 10 add 13 or 23).
- Investigate answer patterns associated with an operation, e.g.

$$\begin{array}{cccc} 16 & 26 & 36 & 46 \\ \underline{-9} & \underline{-9} & \underline{-9} & \underline{-9} \end{array}$$

  and predict answers to related questions (e.g. 86 – 9).
- Investigate and use factors, multiples, prime and composite numbers, including the expression of whole numbers as products of primes.
- Investigate numbers for divisibility by 2, 3, 4, 5, 9, 10 and 11.

B3 use place value to read, write and compare decimal fractions

Possible activities:

- Represent decimal numbers with everyday and structured materials (e.g. code in 2.3 kg on a microwave oven which has 1 kg and 0.1 kg buttons).
- Use decimal fractions to measure in practical situations.
- Use place value to write equivalent forms of numbers to assist with mental computation.
- In realistic contexts, approximate numbers by rounding decimal places, making

judgements about the appropriate number of decimal places.

- Investigate counting and numeration systems from different time periods and the background cultures of the students.
- Use a calculator to investigate place value.
- Explore relationships between spoken and written language and place value in different languages.

B4 interpret common fractions in everyday use and compare fractions for practical purposes

Possible activities:

- Represent fractions as actions on wholes, that is, represent $^2/_5$ of the whole as the action of finding $^2/_5$ of a dish of lollies, a jug of water, or a piece of land.
- Estimate fractional parts of objects and collections (e.g. that piece is about one-third of the salami).
- Find simple everyday equivalents of fractions expressed in words and notation.
- Compare the relative size of simple fractions in realistic situations.
- Use concrete materials and diagrams to investigate the link between division and fractions (e.g. if three people share 2 chocolates, how much does each receive?).
- Use the link between division and fractions to devise a way of expressing a common fraction as a decimal on a calculator.

B5 interpret ratios and percentages in everyday use

Possible activities:

- Interpret percentage language and notation in everyday contexts such as test results.
- Interpret ratios in everyday contexts such as maps and scale drawings.
- Recognise the same ratio in different representations (e.g. 2:3 and $^2/_3$).

B6 use numbers in a variety of equivalent forms and translate between forms

Possible activities :

- Understand that the same number may be represented in a variety of different forms.
- Represent simple common fractions as decimal fractions mentally or with paper-and-pencil.
- Enter simple common fractions into a calculator to find the decimal equivalent, and recognise the calculator decimal equivalents of familiar common fractions (e.g. $^1/_3$ is shown as 0.33333333).
- Use equivalents of common fractions in realistic situations and relate this to percentages (I spelt 9 out of 10 words correctly and I got 18 out of 20 mental calculations right, so I got the same fraction right each time; my percentage right was 90%).
- Investigate and express simple inequalities, using natural language and diagrams (e.g. one-third is more than one-quarter but less than one-half; it is between one-quarter and one-half).
- Recognise the use of negative integers for simple practical purposes (e.g. compare temperatures).

COMPUTATION AND ESTIMATION

Experiences in computation and estimation should be provided which enable children to:

B7 make an appropriate choice of addition, subtraction, multiplication or division of decimal numbers, money and measures

Possible activities:

- Explore practical situations to extend the idea of multiplication to include multiplying by fractional amounts (as in area, scale and rate problems).
- Explore practical situations to extend the idea of division to include dividing by fractional amounts and to relate $m \div n$ to $^{m}/_{n}$.
- Use measuring equipment to undertake investigations leading to statements of equality (e.g. show $4 \times 3 = 2 \times 6$ with a variety of material).
- Play calculator games which lead to number generalisations (e.g. multipliers less than 1 have a decreasing effect and divisors less than 1 an increasing effect).
- Relate mathematical language and symbolism of operations to problem situations.
- Apply distributive properties of operations on numbers (e.g. mentally compute 16 x 7 as 10 sevens add 6 sevens).

B8 remember basic addition and multiplication facts and perform mental computations on whole numbers and money

Possible activities:

- Develop strategies for counting large numbers/collections in practical situations.
- Engage in activities to help memorise basic addition and multiplication facts.
- Extend and practise flexible mental arithmetic strategies developed in band A (see A8).
- Devise strategies to extend mental skills in order to add and subtract two-digit numbers and multiples of 10.
- Devise strategies to extend mental skills in order to multiply/divide single-digit multiples of powers of 10.
- Play 'think of a number' games to improve mental arithmetic skills.
- Investigate numbers for divisibility (e.g. by 2, 3, 4, 5, 9, 10 and 11).
- Build up arithmetic statements of equality and check with peers, e.g.:

$$(4 \times 3) + 10 - 2 = (7 \times 2) + 6$$

B9 use a four-function calculator with memory facility efficiently

Possible activities:

- Use a calculator to perform the four basic operations on any numbers arising in practical situations.
- Practise the skills needed to enter fractions, and use the memory and constant addition and subtraction functions.
- Recognise and adapt to different orders of operation.
- Interpret remainders in the division process and decide whether to round up or down.
- Use simple order of magnitude arguments to check for calculator error.
- Interpret calculator answers which refer to money and measures (e.g. interpret .5 as 50 cents).

- Investigate the development of a variety of calculating devices (e.g. the abacus, the calculator).

B10 estimate and approximate computations on decimals, money and measures

Possible activities:

- Perform approximate calculations using whole number approximations.
- Estimate the upper and lower bounds for the results of computations.
- Estimate answers to problems in context (e.g. will the answer be greater or less than one?).
- Devise and compare strategies for putting an upper and lower bound on the result of calculations (e.g. 47 x 32 will be more than 40 x 30 and less than 50 x 40 , so it will be between 1200 and 2000).

B11 use efficient paper-and-pencil methods to add and subtract decimals, money and measures and to multiply and divide decimals by single-digit numbers

Possible activities:

- Invent and justify paper-and-pencil methods for carrying out computations.
- Use a number line to add and subtract decimal fractions.
- Compare strategies which facilitate the accurate addition and subtraction of decimal numbers for ease and reliability.
- Practise the use of efficient methods to add and subtract decimal numbers and monitor one's own accuracy.
- Adjust units on money and measures in order to add and subtract amounts.
- Work in groups to solve numerical problems by 'guess, check and improve' (e.g. two numbers add to 20, and when multiplied give 96: what are they?).
- Practise the use of efficient and reliable strategies to multiply and divide a decimal number by a one-digit number, monitoring one's accuracy.
- Devise and compare a range of personal paper-and-pencil strategies to facilitate the accurate multiplication of decimal numbers (e.g. repeated addition, repeated doubling).
- Devise personal paper-and-pencil strategies to facilitate accurate division (e.g. progressive sharing) and compare with those of others.

B12 choose computational methods (mental, paper-and-pencil, calculator) and check reasonableness of results

Possible activities:

- Choose appropriate means of completing calculations.
- Judge reasonableness of results (e.g. are the input data reasonable? does the answer make sense in context?).
- Choose and execute appropriate checks on accuracy (e.g. by repeating the calculation in a different order, carrying out rough approximation, and using patterns of final digits).

NUMBER in BAND C

Overview

Students in band C should continue to develop their capacity to represent numbers in a variety of ways and move between representations. Explorations of number patterns and generalisations should also continue to be a central activity during these years (see **Mathematical inquiry**).

The understanding of place value develops over a considerable period of time and should continue to be the focus of learning experiences for students. Decimal place value concepts are necessary for such everyday purposes as ordering numbers, reading scales and measuring small quantities. The important ideas here are those of the ordering and relative magnitudes of decimal numbers. Both of these become more difficult when dealing with very small and very large numbers for which concrete or visual referents are not available.

During these lower secondary years, number will be extended to include the meaning of negative and irrational numbers although, in the latter case, the emphasis should be on numbers like π and square roots which arise out of other contexts, such as measurement. Students should use a number line and begin to develop the notions that numbers form a continuum and that between any two real numbers there is another real number.

They should apply number operations to a wide range of problem situations, developing the executive skills needed to select operations and procedures, and judge the reasonableness of results (see **Choosing and using mathematics**). The emphasis should be on the sensible and realistic use of number skills for practical purposes. Students need to maintain and consolidate their techniques for mental arithmetic, estimation, calculator use and paper-and-pencil work so that they become confident of their capacity to deal with everyday computational situations correctly and efficiently. Some will also extend the types of numbers on which they can operate to include addition and subtraction of negative numbers which arise in realistic settings.

Students in the lower secondary school are approaching the time when they will begin to earn money of their own and take responsibility for managing their own finances. Consequently, social and commercial arithmetic become increasingly relevant.

The use of computing technology should be assumed. Whether the technology chosen is hand-held calculators or computers will depend to an extent on the availability of appropriate hardware and software, but also on pedagogical decisions made by teachers. Consider, for example, an investigation of the return on an investment of $1000 in various interest-bearing accounts. For an account returning 12% per annum calculated monthly, one could calculate the monthly balance step by step over, say, 24 months. If undertaken by hand, the 24 calculations would be extremely tedious and time consuming. Errors are likely to occur and their effect would be compounded. The essential features of the problem are likely to be lost. The same step-by-step calculations could be undertaken by a scientific calculator somewhat more quickly and accurately, although still with considerable tedium. On the other hand, a calculator could be used to determine the return after 24 months in a single calculation. This would avoid the tedium but mask the step-by-step changes and therefore the compounding effect of interest. A spreadsheet, if available, can be used to do the step-by-step calculations with the simple press of a recalculation key, thus giving instant results without masking the dynamic features of the task.

The difficulty, of course, is that it takes some time to learn to use a spreadsheet and the resources must be available. Other uses besides this one would be needed to make the time commitment worthwhile. There are other areas of mathematics, particularly in mathematical modelling, which could benefit from the use of spreadsheets. A positive approach might be to share the time needed to develop the necessary skills across several curriculum areas, such as business studies and computing studies. Regular access to computers is necessary if students are to learn to make sensible choices for themselves among the tools available for dealing with particular situations.

Students should further develop their understanding of the interactions between computational methods and social development and the implications of the computations we do.

Number

NUMBER AND NUMERATION

Experiences in number and numeration should be provided which enable students to:

C1 describe the development of counting and numeration systems in various cultures and relate these to social change

Possible activities:

- Investigate the number-related work of selected female and male mathematicians past and present.
- Investigate numeration systems from a historical and cultural perspective, considering the needs of societies past and present.
- Discuss the significance of the development of zero and place value, and the advantages of a place value numeration system over other systems.
- Investigate and explain a variety of methods for calculating in order to develop a sense of number structure (e.g. compare the two 'standard' subtraction algorithms and those taught in some of the students' countries of origin, considering why they work and comparing them for efficiency of use and ease of remembering).
- Investigate number patterns in nature, art and architecture, considering their aesthetic and utilitarian purposes, and their history.
- Distinguish between separators which are and are not decimal separators (e.g. digital clocks do not use decimals; also for section headings in reports, 6.2 is often before 6.12).
- Investigate the development of the Dewey system for ordering library books.

C2 count and order collections and measures, using 'standard form' for small and large numbers

Possible activities:

- Read, write (in symbols and words) and say numbers of any size.
- Extend strategies for counting large collections and 'count' by decimal parts.
- Order decimal numbers for practical purposes such as shelving library books.
- Order fractions appearing in everyday contexts such as camera shutter speeds.
- Order negative integers for practical purposes such as comparing minimum temperatures of cities around the world.
- Order numbers expressed in standard form for practical purposes such as comparing particle sizes or astronomical distances.
- Compare large and small decimal numbers in everyday occurrence.
- Mark various objects on a scale of powers of ten to make order of magnitude comparisons (e.g. flea, mouse, rabbit, elephant, dinosaur).

C3 distinguish between rational and irrational numbers and understand the significance of recurring and non-recurring decimals

Possible activities:

- Use fractions as mental objects independent of particular representations, that is, use $^2/_5$ as a rational number.
- Represent rational and irrational numbers on a number line.
- Find a decimal number between any two decimal numbers.
- Explore the effect of truncation of decimal places by calculators (e.g. $1 \div 3$ is shown as 0.333333333 and $0.33333333 \times 3 = 0.99999999$, but $1 \div 3 \times 3 = 1$).

- Investigate the effect of calculators working with rational numbers on the precision of calculations.

C4 investigate and use number patterns and relationships

Possible activities:

- Recognise, describe and continue number patterns through activities such as 'guess my rule'.
- Develop simple strategies, such as finding common differences, for determining the rule for number patterns.
- Understand and use factors, multiples, prime and composite numbers, including expressing whole numbers as products of primes where doing so serves a purpose.
- Recognise and represent squares, square roots and cubes, and odd and even, figurate, palindromic and Fibonacci numbers and understand their aesthetic dimension.
- Use $<$, $>$, $\geq$, $\leq$, $\neq$, as convenient devices for expressing inequalities.

C5 use a variety of equivalent forms of numbers (e.g. decimal fraction, common fraction, standard form, percentage) and convert between them

Possible activities:

- Represent decimal and common fractions (e.g. using concrete materials, diagrams, and calculators).
- Use simple common fractions and decimal fractions in practical situations such as interpreting recipes or other instructions.
- Translate between numbers expressed in index notation and other representations.
- Express numbers in standard form (scientific notation) using positive and negative powers of ten and read displays on scientific calculators.
- Represent common fractions in alternative fractional forms and as decimal fractions, choosing appropriate strategies (mental or with paper-and-pencil for simple equivalences, and in other cases, calculator).
- Relate common fractions and ratios to percentages and interpret percentages and ratios appearing in everyday contexts.
- Recognise and use different representations of the same proportion (e.g. 2:5 and 4:10).

COMPUTATION AND ESTIMATION

Experiences in computation and estimation should be provided which enable students to:

C6 make an appropriate choice of addition, subtraction, multiplication or division of rational numbers for a given context

Possible activities:

- Through practical experiences extend meanings for the four basic operations to include:
  division as comparison (ratio);
  multiplication and division by rational numbers of any size;
  addition and subtraction of negative numbers.
- Choose from amongst the four operations of addition, subtraction, multiplication and division the correct one to use in a particular situation.
- Through practical experiences (e.g. with concrete materials, diagrams or calculators) represent powers of numbers.
- Relate mathematical language and notation of operations to problem situations.
- Use commutative, associative and distributive properties in context.
- Use parentheses and order of operations in meaningful contexts such as stocktaking and inventories.

C7 apply ratio, proportion and percentage in everyday contexts

Possible activities:

- Use ratio and percentage in familiar situations and relate mathematical language and notation of percentage and ratio to problems.
- Relate ratio to division and fractions and express percentages as ratios and fractions.
- Compare the use of the common fractional notation to describe 'parts of a whole' and 'ratio' and the implications of each model for operations on fractions (e.g. when fractions are used to model ratio situations such as mixtures, the standard addition model does not apply.)
- Compare numbers and quantities and explain relationships, using ratio and proportion language.
- Recognise and informally use direct and inverse variation in practical situations.
- Convert currencies from one to another, and research the movement of currencies over a short period, deciding (retrospectively) the day which would have provided the best yield for a given exchange.

C8 remember basic addition and multiplication facts and perform mental computation on decimals, including money and measures

Possible activities:

- Extend strategies for counting large numbers/collections (e.g. systematically count the takings for a fete or the people attending an event).
- Extend and practise flexible mental arithmetic strategies, including those for: adding and subtracting 2 two-digit numbers; adding and subtracting multiples of 10; adding and subtracting simple fractions; multiplying/dividing by single-digit numbers (e.g. multiply by 9 by replacing 9 by 10 – 1 or multiply by 4 by doubling twice); and multiplying short series of single-digit numbers.
- Give and receive change by 'counting on'.

C9 use a scientific calculator efficiently

Possible activities:

- Extend calculator techniques to evaluate expressions involving four basic operations on any numbers.
- Make use of the constant function.
- Use memory and bracket facility to plan calculations and evaluate expressions including substitution of any numbers into formulae.
- Discuss the effect on precision of the fact that calculators work with rational numbers.

C10 estimate and approximate computations on any numbers

Possible activities:

- In situations where it makes sense to do so, approximate numbers by rounding and reading significant figures or decimal places.
- Anticipate possible effects of calculator rounding errors.
- Approximate to check the validity of any calculation, including those involving direct and inverse proportion, and scales and rates.
- Express numbers in standard form and use rules for operating on indices to make order of magnitude checks for accuracy.
- Decide whether an answer in a given situation must be accurate or whether an approximation will suffice, and explain reasoning.
- Devise and compare strategies for quick and effective estimation of percentages (e.g. estimate 12.5% discount of the cost of the garment).
- Devise and compare strategies for forming reasonable estimates of numbers in collections (e.g. crowds, pages in a book) based on everyday reference points and counting and computation.
- Devise and compare strategies for forming order of magnitude estimates of numbers not easily obtained by measurement or counting (e.g. number of pies eaten in Australia this year).
- Decide appropriate range of 'good' estimates/approximations in context and the upper and lower bounds of estimates.

C11 use efficient paper-and-pencil methods to add, subtract and multiply whole numbers and decimal and common fractions, and to divide by single-digit numbers

Possible activities:

- Devise a range of personal 'back-of-envelope' strategies to add and subtract decimal and fractional numbers, and compare for efficiency.
- Practise efficient methods to add and subtract decimal and fractional numbers arising in practical contexts.
- Develop a range of personal written strategies to adapt single-digit multiplication to multiplication by two-digit numbers (e.g. use the distributive property — eighteen 27s is ten 27s add eight 27s) and compare for efficiency.
- Practise the use of efficient methods to multiply and divide decimal numbers by single-digit numbers.
- Practise the use of efficient methods to multiply decimal numbers by two-digit numbers.
- Calculate simple ratios and fractional and percentage changes in practical contexts.

C12 choose computational methods (mental, paper-and-pencil, calculator, computer) and check reasonableness of results

Possible activities:

- Determine whether an exact or approximate solution is needed in the context.
- Choose and justify appropriate means of completing calculations.
- Judge reasonableness of results and choose and execute appropriate checks on accuracy.

C13 interpret the language of and deal with matters of personal finance

Possible activities:

- Make 'rough' estimates of the cost of a shopping basket (e.g. I have $10.50 with me; will it be enough?).
- Make 'rough' price comparisons, for example, by separating items into those which cost more than a cent per gram or less than a cent per gram.
- Identify best buys using practical and realistic strategies for the context (e.g. coffee costs $5.48 for 250 g and $11.65 for 375 g — to decide which is the better buy, doubling $5.48 (rounded to $5.50) is sufficient; to calculate unit prices is inefficient).
- Devise and compare strategies for quick and effective estimation of percentage discounts (e.g. estimate 12.5% discount off the cost of the garment).
- Calculate profit and loss on purchases, considering 'mark-ups' and 'mark-downs'.
- Research and report the facts about certain consumer items considering preferences, price, quality, discounts, lay-bys, warranties, term payments, savings schemes, insurance, etc.
- Calculate earnings for various time periods including calculations of gross pay, compulsory and voluntary deductions and net pay.
- Research the obligations of employees and employers under specific industrial awards, focusing particularly on matters affecting youth.
- Estimate interest charges for short periods (e.g. I borrowed $870 at 2% per month; which is the best estimate of the interest charges for six months: $16, $40, $110, $400?).
- Use a calculator or spreadsheet to compare daily or monthly interest rates with effective annual rates (e.g. Bankcard interest rates).
- Compare different types of saving and cheque accounts available, comparing minimum balances, interest earned, charges, etc.

NUMBER in BAND D

Overview

During band D, students should clarify their understanding of the various types of numbers and ways of representing them. They should be helped to develop an appreciation that people developed these various numbers in order to satisfy purposes which were practical and also aesthetic.

In each case, numbers were developed in order to fulfil a desire for a system of numbers that formalised our intuition about the physical world and was in some way coherent and complete. Thus negative and rational numbers are a natural progression from subtracting and dividing natural numbers. If we feel that we ought to be able to cut any piece of string into three equal pieces and that there ought to be numbers which correspond to the result of doing so, then we need numbers such as one-third and two-thirds. Students may investigate the history of irrational numbers and reflect on whether the name reflects how people thought about them at the time. They should also consolidate the notion that the real numbers can be identified with a line, and that the rational and irrational numbers are dense in the reals (that is, any interval on the real number line contains a rational number, and an irrational number).

Those intending to proceed to the study of mathematics and certain related fields will probably need to recognise and use various representations for complex numbers (e.g. vector, matrix, polar, $a + ib$). These representations should be linked with work in the **Space** strand.

All students should be able to compute fluently with simple rational numbers, square and cube roots and integral powers to numerical bases. Calculators reduce but do not completely replace the need to simplify surdic expressions. For some students it will be appropriate to develop the basic terminology and rules, such as that for rationalising denominators.

In the post-compulsory years many students are in the transition between the learning environment of the classroom and the world of work. In the former, part-marks are often given to encourage students and to reward them for what they do know. In the latter, accuracy may often be vital and 'a good try' simply not good enough. For such tasks, reliability is of prime importance and 'part-marks for method' may make little sense. Therefore, whether their courses draw mostly from band C or band D these students should be expected to recognise situations in which absolute accuracy is essential and to ensure that they achieve it.

Students should also study error propagation, considering especially those which may arise in the use of calculators and computers. For example, arithmetic limitations in certain calculators will produce certain types of error. A truncating calculator will show:

$$12345678 + 0.4 + 0.3 + 0.3 = 12345678$$

$$\text{but } 0.4 + 0.3 + 0.3 + 12345678 = 12345679$$

Students should be conscious of the potential for such errors to occur and the effect to be cumulative (see **Measurement**, D1). They should plan calculations (e.g. rearrange expressions) to increase accuracy of computation.

BEST COPY AVAILABLE

The study of patterns should continue to underpin number work in band D. Students should observe common patterns underlying apparently different situations (e.g. paper folding, growth and decay, interest charges). Their representation of these patterns should include algebraic (including recursive), graphical and spreadsheet forms.

Students should develop and use models based on sequences and series in order to better understand situations and make predictions about them. Sequences and series have immediate applications in financial mathematics and are also important for further study in mathematics. Some study of infinite series will be appropriate for most students and some may make a more formal study of the notion of limits, including the study of different kinds of behaviour of some simple iterative processes.

Students should also be familiarised with a range of techniques for making systematic counts (e.g. tree diagrams, permutations, combinations).

Finally, all students need to understand the implications that debt and the cost of borrowing money have for them. While not all students need to fully understand the technical language and mathematical principles which underlie personal and commercial financial transactions, they should have experiences of making decisions based on financial calculations in situations which are personally relevant to them.

NUMBER AND NUMERATION

Experiences in number and numeration should be provided which enable students to:

D1 identify, derive and compare properties of sets of numbers

Possible activities:

- Explain the 'need' for irrational numbers (e.g. we feel there 'ought' to be a number to correspond to the length of the diagonal of a square of side 1, but we can show that the only possible value for the diagonal, $\sqrt{2}$, is not a rational number).
- Compare subsets of the real numbers and identify differences and commonalities (e.g. identify what is common and what is different between natural numbers and integers, fractions and decimals, rationals and irrationals).
- Apply infinite geometric series to examine results such as 0.9 recurring = 1.
- Prepare concept maps or Venn diagrams to show the relationship between sets of numbers.
- Make a study of the properties of integers (e.g. the fundamental theorem of arithmetic, infinitely many primes, ...).
- Describe and compare the field properties that hold in different systems (e.g. natural, rational, real, and complex numbers).
- Identify properties of number systems which also hold in other systems (e.g. polynomial functions that have rational coefficients, system of matrices, ...).
- Identify the 'need' for complex numbers (e.g. generate solutions of quadratic equations in which the discriminant is negative).

D2 represent numbers in a range of forms, and use in solving problems

Possible activities:

- Use indices to model phenomena (e.g. relating to bouncing balls, illumination, irradiation, machine efficiency, bacteria count, photo reduction/enlargement).
- Represent numbers in exponential and logarithmic forms and relate the two forms.
- Investigate and report on various forms of scale (e.g. linear, linear logarithmic, decibal, Richter, piano keyboard).
- Make a study of representations for certain numbers (e.g. various approximations for π or e).
- Represent complex numbers using various methods (e.g. vector, polar, algebraic, on the Argand diagram,...) and select an appropriate form for a particular problem.
- Demonstrate consistency between operations on the various representations of complex numbers.
- Interpret and use absolute value notation with real and complex numbers (e.g. to express magnitudes, in inequalities).
- Represent numbers in various bases (e.g. explore the outcomes of calculations in binary, hexagonal or octal using a scientific calculator).

D3 develop and use models based on sequences and series

Possible activities:

- Determine parameters (e.g. specific terms, sums, general terms) through investigations of various sequences and series such as arithmetic, geometric, Fibonacci, logarithmic, etc.

- Express sequences recursively (e.g. write a recursive expression for $1, {}^1/_4, {}^1/_9, {}^1/_{16}, \ldots$).
- Apply models based on sequences and series to problems (e.g. calculation of annuities, determining the least number of weighings needed to determine the lightest object in a batch, population growth calculations).
- Compare and contrast numerical patterns (e.g. logarithmic and exponential sequences, triangular numbers and Pascal's triangle).
- Explore end game strategies (e.g. to score 501 in darts).
- Investigate situations which can be represented by an infinite series (e.g. cut a unit square in half, halve one half, halve a half of the half, etc.) leading to the notion of a limit of the sum to infinity.
- Examine some specific infinite series in order to obtain simple important understanding such as the relationship between the size of u_n and the behaviour of Σu_n, e.g. examine $\Sigma({}^1/_{\sqrt{n}})$, $\Sigma({}^1/_2)^n$ and $\Sigma({}^n/_{2n+1})$
- Examine some simple iterative functions of the form $x_{n+1} = f(x_n)$, numerically and algebraically in situations which illustrate different kinds of behaviour (e.g. convergence or otherwise).

COMPUTATION AND ESTIMATION

Experiences in computation and estimation should be provided which enable students to:

D4 estimate numbers and make approximate computations

Possible activities:

- Create a personal budget and adjust estimates as variables alter (e.g. increase in petrol price, increase in wage, increased taxation level).
- Estimate solutions to equations on the basis of graphical form (e.g. using graphical calculator, halving the interval method).
- Make order-of-magnitude calculations to check computations involving very large or very small numbers (e.g. use figures from the financial section of the newspaper or from scientific literature).
- Estimate upper and lower bounds for infinite sequences and series or definite integrals.
- Formulate and make order-of-magnitude estimates for Fermi problems (i.e. problems of the kind: how many taps are there in a neighbourhood, suburb or city? how much newsprint is used each day in Australia and how many trees are needed to produce it?).

D5 choose and use appropriate formulae, counting and computational strategies and technologies

Possible activities:

- Develop and compare calculator routines for evaluating expressions (e.g. compound interest formulae).
- Develop and use efficient calculator strategies for computing very large or very small numbers.
- Make a systematic study of potential errors inherent in the use of certain calculators, including the accumulating effect of errors.
- Plan and execute calculator routines to avoid or minimise errors.
- Make a systematic study of techniques for dealing with ratios.
- Use various techniques to enumerate all the possibilities in a problem situation (e.g. tables, tree diagrams, iteration formulae, ...).
- Use standard notations, procedures and formulae for counting (e.g. factorial, ${}^nP_r$, ${}^nC_r$, Σ) in problem situations.
- Examine relationships between formulae for combinations, Pascal's triangle and the binomial theorem.
- Write a computer program to perform a calculation outside the scope of conventional calculators (e.g. expressions involving large factorials).

D6 understand and apply aspects of personal finance and business-related arithmetic

Possible activities:

- Investigate the effects of various inflation rates (e.g. on wages, home loans, savings).
- Make financial decisions based on calculation (e.g. early repayment of personal loan, lump sum payments towards house mortgage).
- Compare different loan packages available to purchase items of interest to students, considering a range of variables (e.g. interest rates, application charges, stamp duty,

total cost of loan, methods of repayment, default conditions, need for guarantor, loan insurance).

- Use a calculator or spreadsheet to investigate the long-term effect of making fortnightly or monthly payments on a loan, given the choice between fortnightly or monthly calculation of interest.
- Use a calculator or spreadsheet to solve financial problems which require organisation of information and analysis of change over time (e.g. in 1972, Sarah was born and her aunt placed $100 in the bank for her; the money remained in the bank account earning interest at an annual rate of 6% calculated six monthly; what should Sarah expect to receive on her 21st birthday?).
- Interpret media reports involving price indices and economic statements.
- Undertake calculations and make decisions relating to investments (taking into account, for example, depreciation, exchange rate variations, brokerage fees, bank charges, taxes, ...).
- Understand and apply aspects of time series (e.g. three-year averages relating to retirement benefits payments, or share purchase plans).

Measurement

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Purpose and scope

Measurement is the quantification of some feature of objects, people or events and underlies many of the descriptive statements we make. We may with reasonable confidence measure how tall people are or how fast they run. With somewhat less confidence we may measure their mathematics achievement and possibly even how nervous they felt before testing began. Most of us measure or interpret measurements almost every day of our lives. Some are informal but quite practical like, 'We have just enough water (fuel, food) left to last another day'. Others may be more formal, such as estimating levels of chloro-fluorocarbon emissions.

Knowledge of the measures in common use and skill in making measurements are of practical use every day. Understanding the basic principles of measurement and the confidence that can be placed in various kinds of measurements also assists people to make sensible interpretations of the results of a wide range of inquiries. The ability to make informed judgements about measures is important when actual measurement is difficult or awkward or tolerances are large. It is of direct practical benefit for estimating (e.g. The burglar who ran away was about 1.8 m tall) and for judging the reasonableness of a result (e.g. Could the average height of women in Australia really be 217 cm?). For the majority of people, most of the measurements they make are likely to be based on such judgements.

Measurement involves the application of number to spatial (and other) qualities of objects and events. Therefore spatial, numerical and measurement concepts generally develop together. Examples include: the links between tessellations (**Space**) and area (**Measurement**); and links between scale drawing (**Space**), multiplication, fractions and ratio (**Number**) and rates and ratios (**Measurement**). Probability is our way of measuring chance and early work on chance bears a close relationship to work with other measurement concepts. The data we handle are usually measurement data and this is reflected in **Chance and data**, **Algebra** and **Choosing and using mathematics**. Practical measurement activities should occur across the school curriculum. 'Extra-curricular' activities, such as concerts and excursions, also provide a rich source of measurement activities. This should assist students to understand principles of measurement and develop the skills they will need at home, in school and in work.

This strand is described under the subheadings: *Measurement and estimation*, *Indirect measurement* and *Approximation, change and the calculus*.

Measurement and estimation

Students should develop the concepts and skills involved in measuring the basic attributes of length, area, volume and capacity, mass, angle, time and temperature and derived measures.

The fundamental idea which underlies measurement is the comparison of one thing with another according to some specified feature. For example, when we say that a child weighs 21 kilograms we use numbers to compare the child's mass with a standard object which has a mass of 1 kilogram; we have used the kilogram mass as a *unit*. The process of measuring involves choosing a unit (e.g. handspan), repeating the unit until it 'matches' the thing to be measured according to the attribute of interest (e.g. length), and counting how many of the units it takes to make the match. In practice, we need not always perform a physical match because people have developed a range of techniques to simplify this process, but the underlying principle remains the same.

While we can *count* discrete quantities such as people or pages exactly, *measurements* are inexact and exist within limits which are specified by the unit in use. Thus, when we say that an opening is 106 cm wide we mean that its width is closer to 106 cm than to 105 cm or 107 cm. If we want a more accurate measure then we choose a smaller unit, perhaps a millimetre. The size of the unit and the level of care to be taken depend on the reason for measuring. A building may be measured with considerable precision (using a unit like a millimetre) for detailed engineering work, but with less precision (using a unit like a metre) as a basis for estimating the amount of paint needed to cover it.

Students will need to develop the following ideas:

- some objects work better as units than others (e.g. when measuring area use area units, and the units should be able to fit together without overlaps or gaps);
- there is an inverse relationship between the size of the unit chosen and the number of units needed to measure a given object;
- all units are arbitrary in size, but in order to compare two things the same unit is necessary for each;
- the need for greater precision of measurement necessitates the use of finer units;
- standard units are used when the measurement must be transported or communicated.

This will require a great deal of practical experience with a variety of non-standard units and should not be hurried.

In order for students to learn to measure and estimate quantities a range of experiences is necessary. For capacity, children will need to:

- become aware that 'how much it holds' is one of the attributes of a jar and that we call it the capacity of the jar;
- realise that we can compare and order jars by their capacity either by pouring from one to the other or by using some object as a unit and counting how many it takes to fill each jar;
- develop the skills needed to choose and use a unit directly (e.g. fill each jar using bottle tops as a unit) or use measuring equipment (e.g. fill each jar by filling with water, decanting into a measuring jug and reading the scale);
- develop a reliable sense of how much, for example, a litre is, and judge the capacity of containers with a degree of confidence.

A similar set of experiences is needed for each attribute (e.g. length, mass, time, and so on), although students are likely to be at different stages of development with different measures.

Indirect measurement

Indirect measurement is necessary when the direct comparison or measurement of quantities is either impossible or impractical. It may take several forms. We can determine the area of a rectangle directly by covering it with unit squares and counting the number of squares or indirectly by measuring the lengths of two adjacent sides and finding the product (making appropriate adjustments of units). In addition to measurement formulae, we may use Pythagoras' theorem, known ratios (including trigonometry) and rates, scale drawings and models.

To directly measure the thickness of a sheet of paper would require equipment of some considerable precision (and possibly expense) but we might measure it indirectly by measuring the thickness of a thousand sheets. In this way we can optimise the use of the available equipment, obtaining a more precise measure than the equipment can provide directly. We may also reason on the basis of familiar or known quantities to estimate unknown quantities, for example, estimating the size of today's crowd by comparing it with last week's.

The concepts underlying indirect measurement are often sophisticated and their development within mathematics will rely on a very firm foundation of spatial and number concepts. Therefore, while indirect measurement first appears as a separate subsection in band C, the groundwork will be laid earlier.

Approximation, change and the calculus

For linear relationships (e.g. the cost of apples given the price per kilogram), one variable (i.e. cost) changes at a constant rate with respect to the other (i.e. the weight). Many relationships, however, show much more complex patterns of change where the rate itself varies. Examples of rates which may vary over time include mortality and fertility, radioactive decay, inflation, velocity and acceleration. To interpret a statement such as 'inflation is falling' requires an understanding of the difference between a change in state (in this case, a change in prices) and a change in rate (in this case, a change in the rate at which prices are increasing). It therefore requires that we see 'rate of change' as a variable.

Investigating the rate at which one variable changes as another varies may deepen our understanding of their relationship to each other and can provide new ways of finding important information about it. Fundamental to the development of this understanding is the mathematical idea of *instantaneous rate of change* and its graphical interpretation. This concept forms the basis of the differential calculus.

In other situations, we may need to find both *estimates* of and *exact values* for areas, volumes and other magnitudes associated with surfaces or solids of arbitrary shape. Such measurement problems arise naturally in a number of contexts and a variety of methods of dealing with them have been used over time. One such method involves the use of estimates by means of simpler shapes. A simple illustration of this comes from estimating areas of irregular shapes by covering them with square grids. As the mesh of the grid is reduced a better estimate for the total area is obtained. This basic notion led, several hundred years ago, to the development of techniques for determining exact expressions for such magnitudes by calculating the limiting values of these estimates as the mesh size approaches zero. This is the conceptual basis for the *definite integral*. An understanding of this essential idea allows one to model many other situations in which a total quantity of something (e.g. electricity, population) can be represented as a definite integral.

These two fundamental concepts of the calculus are of considerable significance in formulating mathematical models of a host of problems in which measuring rates of change or total amounts of quantities are essentially involved.

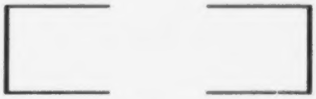
MEASUREMENT in BAND A

Overview

People carry out a great deal of 'intuitive measurement' which does not involve units or numbers, for example, 'Can I jump over the stream?', or 'Am I speaking quietly enough?'. This kind of judgement is essential to our being able to carry out the quite complex activities of talking and moving about in space. Young children will come to school with considerable experience of this kind and will usually have developed intuitive notions of more, of less, and of equal amounts and some of the language of comparison.

They will, however, normally require a great deal of experience before they develop comparative language fully. For example, the difficulty children have with bipolar comparatives may be underestimated. They are likely to understand some pairs well before others (e.g. multidimensional pairs such as big–small, before uni-dimensional pairs such as high–low) and 'positive' members of pairs before 'negative' members (e.g. tall before short).

The subtleties of comparative terms will be, to an extent, culture specific. For example, people in North East Arnhem Land talking of a big fish would use their hands to demonstrate the width, whereas Anglo-Australians would use their hands to show the length. The potential for confusion is clear! Attempting to distinguish between the width and length of the hall below indicates just how subtle and idiosyncratic many such distinctions are.



Children will need to experience the use of such pairs as 'wide' and 'narrow' in different contexts, from ribbons to roads to openings, and gradually deal with some of the subtleties of common usage.

During band A, activities should be structured to help children become aware of various attributes of objects and events and to develop the idea of comparing and ordering by quantity. The associated language should develop in use. The more technical language (e.g. perimeter, circumference) is not expected during band A, but children should have the opportunity through informal experiences to lay down a foundation upon which these can be based. Thus they should investigate the 'distance around' and the 'amount of tape needed' long before the technical terms are expected.

Many experiences will not require counting but rather the direct comparison of quantities. Initially children will use units of length, area, capacity, mass and time informally. They should begin to understand the idea of the repetition of a unit without overlaps and gaps and make statements about measures using the language of betweenness (e.g. The room is between 6 and 7 strides wide).

The development of estimation skills should be a high priority and children should make statements about the confidence they hold in their estimates. A young child might estimate a wall to be 7 m wide but claim to be 'absolutely certain' that the wall is between 4 and 10 m wide and 'pretty sure' it is between 6 and 8 m. As they discuss their work, the language of approximation should be clarified (e.g. almost, not quite, a bit less than).

Initially, young children will use standard units in much the way they use non-standard units. As they mature they should come to understand the usefulness of standardising some units for recording and communication purposes.

Measurement activities provide rich contexts for counting and fractional concepts but, at the same time, depend upon sound spatial concepts. Careful integration of measurement with other strands is essential. Some measurement concepts can be expected to develop earlier than others and the scope statements reflect this. For example, during band A children will usually begin to use equipment for measuring length but not angle.

MEASUREMENT AND ESTIMATION

Experiences in measurement and estimation should be provided which enable children to:

A1 recognise the attributes of length, capacity, mass, area, volume, time, temperature and angle

Possible activities:

- Through a variety of practical experiences in cooking, drawing and constructing, shopping, and so on, develop an awareness of attributes:
  - length (e.g. the streamer won't reach from corner to corner);
  - area (e.g. will the paper be enough to cover the book?);
  - capacity (e.g. can we fit the nuts into the box?) and volume (e.g. which takes up more room?);
  - mass/weight (e.g. can you lift the box?);
  - angle (e.g. turn a sharp corner);
  - time (e.g. how long will it take?).
- Use everyday language of comparison to describe attributes of objects and events such as:
  - length (e.g. long, wide, short, narrow, all round, curved, path);
  - area (e.g. cover, all over, surface, top, spread);
  - capacity (e.g. full, empty, holds a lot, not much room) and volume (big, small, takes up a lot of room);
  - mass (heavy/light, hard to push);
  - angle (turn, corner);
  - time (today, soon, long ago, playtime, September, and then).
- Investigate situations which assist children to distinguish the 'distance around' a shape (e.g. the length of the fence) from the amount of surface enclosed (e.g. the area of the lawn).
- Make a variety of different shapes of the same measure (e.g. use a piece of string to explore various shapes with the same distance around; use plasticine to explore different shapes with the same mass and volume).

A2 make direct comparisons of, and order, objects and events using length, capacity, mass, area, volume, time, temperature and angle

Possible activities:

- Develop the language of comparison (more, less, long, short, tall, taller, tallest, etc.) in a wide range of practical contexts.
- Compare and order collections by the number of things in the collection.
- Using the attributes of length, area, mass and capacity, compare and order objects by direct comparison (without counting).
- Recognise that when ordering objects by size, attention to different attributes may lead to different orders (i.e. for different shape containers, height order may be different from capacity order).
- Compare and order events by passage of time (e.g. order children by direct comparison of time taken to complete a race; order events in a day).
- Compare angles and turns identified in the environment.
- Compare and order angles (e.g. cardboard cutout representations) by superimposition.

A3 recognise that units are required for measuring and choose units appropriate to the task at hand

Possible activities:

- Understand that a unit must have the same quality as the attribute to be measured; thus area units are used to measure area and mass units to measure mass.
- Compare quantities by repetition of measurement units, without counting (e.g. rotate through containers, putting a cupful of sand in each container each round; the one filled first is smallest, the one filled next is next smallest, ...) and with counting.
- Investigate the relationship between the size of the unit chosen and the number of units used to measure an object.
- Use the idea that comparing measurements requires a common unit (e.g. 6 hands cover my book and 22 bottle tops cover yours but that doesn't mean that your book is bigger because ...).
- Recognise the need for estimates of part units leading to an intuitive idea of fractions.
- Engage in activities in which there is a need to communicate measurements (e.g. shop for the tape needed for Easter bonnets) in a standard way.

A4 use non-standard units for the measurement of length, area, capacity, volume, mass, time and angle

Possible activities:

- In practical situations count non-standard units to measure length, area, capacity and mass and use the language of 'betweenness' to describe the measures (e.g. Terry is between 8 and 9 handspans tall; the jug holds between 9 and 10 scoops).
- Relate tiling (or tessellation, see **Space**) to area measurement, investigating various non-standard units of area and judging the most suitable units for covering shapes with no gaps and no overlaps (e.g. is the circle or the triangle a better shape for an area unit?).
- Use rhythmic processes to measure time informally (e.g. she threaded the needle in 5 beats, and he threaded it in 6 beats).
- In practical activities describe turns in half, quarter and full turns, and angles as greater than, equal to or smaller than a right angle.
- Make own devices for measuring.

A5 use everyday standard units and measuring equipment to measure length, capacity, mass and time

Possible activities:

- Begin to use the basic units for length (metre and centimetre), capacity (litre and millilitre), and mass (kilogram) and common units of time (seconds, minutes, hours, days, weeks, months and years) in practical situations.

- Make and calibrate measuring equipment to measure length (in metres and centimetres) and capacity (in litres and 100 mL).
- Begin to use measuring equipment:

  length (e.g. ruler, tape measure, trundlewheel in centimetres and metres);
  capacity (e.g. measuring jugs, calibrated in litres and 100 mL);
  mass (e.g. scales calibrated in 100 g and kg);
  time (e.g. digital clock and analog clock).
- Investigate to find that 100 cm = 1 m and 1000 g = 1 kg and 1000 mL = 1 L .
- Develop strategies for measuring lengths which are longer than the measuring equipment (e.g. use a metre stick with centimetre markings to measure a table which is more than one metre long).
- Choose appropriate units for measuring length, recognising that the choice of unit depends on the purpose for which the measurements are being made.

A6 make estimates of length, capacity, mass, area and time

Possible activities:

- Estimate to order objects by size using the attributes of length, area, capacity and mass (e.g. compare three books of different shape and arrange in order of estimated size — from largest to smallest area), and check and improve estimates (e.g. use a unit to cover books).
- Estimate measures using non-standard units and expressing with 'between' statements (e.g. I think the wall is between 6 and 7 armspans wide; I think that lasted between 3 and 4 minutes).
- Relate familiar objects to standard measures (e.g. a matchbox is about a centimetre deep; find five different containers all holding about one litre).
- Estimate to compare objects to standard measures of length, mass and capacity (e.g. compare the mass of an object with a kilogram standard by lifting it and decide 'I think this weighs more than a kilogram'; mark a point a metre away, check estimate and try again in a different direction; the bucket holds more than a litre but the cupcake holds less).
- Begin to develop skills in estimating lengths, heights and distances (in centimetres and metres), capacity (in litres) and mass (in kilograms).

MEASUREMENT in BAND B

Overview

During the upper primary years, children should continue to develop their understanding of the nature and uses of measurement and to improve their measurement skills. Many of the activities will be similar to those of the early primary years and practical work, together with the careful development of comparative language, will continue to be important.

Children should be encouraged to reflect upon and collaborate in making sensible choices about:

- which qualities should be measured for the task at hand;
- which units to use (that is, how accurate you need to be);
- the measuring tool which is most appropriate.

They should discuss their approaches with each other and with their teacher in order to improve upon them.

Their understanding of the use of units should develop considerably during these years and they should be helped to articulate generalisations about units. For example, they may say 'It took 19 of these squares to cover the book, but it took only 12 of the rectangles. That was because the rectangle was bigger than the square. It always works that way. The smaller the unit, the more it takes to cover a shape'.

Children should also be able to distinguish between situations in which non-standard units may reasonably be used and situations where a standard would be helpful. They should understand that standard units are no more correct or precise than non-standard units, but using units which everyone shares has advantages for recording and communicating measures.

By the late primary years, most children should have a good feel for the size of millimetres, centimetres and metres and the associated square and cubic measures, litres, kilograms and seconds and minutes and be able to use these in making estimates. They should recognise that estimation is not simply guessing but rather informed judgement, and efforts to improve estimates should be made explicit.

Through investigations children should generalise some of the relationships between measures which lead to simple mensuration formulae (e.g. that the area of a rectangle is the product of its height and width) and use these in practical situations. *They need not commit these formulae to memory during band B, the focus instead being on an understanding of the meaning of perimeter, area, and so on, and of the types of problems to which these mathematical ideas may usefully be applied.* Activities should be provided which assist children to develop an understanding of the effect of scalar changes (e.g. doubling all the dimensions) on length, area and volume. They should also recognise that, for figures with different shapes, perimeter and area are not necessarily related; that is, one rectangle may have a larger area than a second rectangle but have a smaller perimeter.

MEASUREMENT AND ESTIMATION

Experiences in measurement and estimation should be provided which enable children to:

B1 measure, compare and order objects and events using length, capacity, mass, area, volume, time, and angle

Possible activities:

- Directly compare and order objects according to length, capacity, mass, surface area, volume, and angle.
- Investigate various non-standard units of area, relating tiling (or tessellation, cf. **Space**) to area measurement.
- Starting with a basic tessellating shape (e.g. rectangle, triangle) adapt it to make different tessellating shapes of the same area.
- Count non-standard and standard units to give a measure of length, area, volume, capacity, mass, angle and time and use the language of 'betweenness' to describe the measures.
- Compare the ordering of objects by different criteria (e.g. we ordered by height first and then by capacity and the orders were different).
- Order circles according to diameter and distance around a perimeter in order to investigate relationships (e.g place 'sticky tape' around the edge of various lids, cut and compare order of perimeters with order of diameters).
- Make estimates of part units (e.g. measuring the area of a circle with a square centimetre grid suggests the need to consider part units).

B2 choose appropriate units for tasks by considering purpose and the need for precision and/or communication

Possible activities:

- Reach consensus within a group about which attributes need to be measured to deal with the situation at hand (e.g. to find who slept latest we need a measure of the time of waking; to find who slept longest we need a measure of elapsed time).
- Judge the suitability of units for measuring area and volume so that the units can cover or fill the object to be measured with no gaps and no overlaps.
- Judge which unit will give the most accurate measurement (e.g. these triangles are smaller than the squares so they will give a more accurate measure of the area of the lid).
- Select appropriate unit size for the task at hand and justify choice (e.g. to the nearest quarter hour is best because …).
- In practical situations, decide which unit will suffice (e.g. the fabric is sold to the nearest centimetre so …).
- Investigate body measures (e.g. cubits) and the use of old measures in present society.
- Recognise the advantages of metric units and the reasons for Australia's change to the SI system.

B3 recognise standard metric units of length, capacity and mass and use common metric prefixes to relate units to each other

Possible activities:

- Make and calibrate measuring equipment to measure length (in centimetres and metres), capacity (in litres and 100 mL lots) and mass.

- Use the most commonly used standard units for length (metres and derivations — millimetres, centimetres and kilometres), capacity and volume (litre, millilitre, cubic centimetre, cubic metre), mass (gram and kilogram), time (seconds, minutes, hours, days, weeks, months and years) and area (square centimetres and square metres).
- Use the relationship between litres and cubic measures of volume.
- Explain the relationship between a standard unit and its derivations (e.g. one thousand metres is a kilometre because kilo always means thousand).

B4 use standard techniques and tools to measure length, capacity and volume, mass, area, time and angle

Possible activities :

- Use measuring equipment in practical situations:
  length (ruler, tape measure, trundle wheel in millimetres, centimetres and metres);
  capacity (measuring jugs calibrated in millilitres and litres);
  mass (balance beam, scales calibrated in 10 g, 100 g and kg);
  time (digital clock and analog clock);
  angle (full circle protractor).
- Use centimetre-square grids to measure the area of irregular shapes for practical purposes (e.g. estimate the area of a state or country from a map of the country; find the area of a person's hand to the nearest square centimetre and use to predict body surface area).
- Measure the volume of irregular objects by the displacement of water.
- Develop strategies for measuring lengths which are longer than the measuring equipment.
- Choose appropriate units for measuring length, area, capacity, mass, angle and time, recognising that the choice of unit depends on the purpose for which the measurements are being made as well as on the size of the object.
- Place special celebration days of class members on a calendar (including birthdays, name days, festivals and religious holidays).
- Investigate various ways people have measured and do measure time, and different calendars, past and present.

B5 place objects in order by estimating their relative sizes and make estimates of length, capacity and volume, mass, area, time and angle

Possible activities:

- Judge the size of common standard units (i.e. find lengths of approximately 1 mm, 1 cm and 1 m, areas of about 1 cm^2 and 1 m^2, capacity/volumes of about 1 L and 1 m^3 and masses of about 1 kg).
- Estimate, measure to provide feedback, and estimate again in order to improve judgement about measures of length, capacity and volume, mass, area and time.
- Discuss factors which can distort estimates (e.g. time passes slowly when …) including visual illusions.
- Make a graph of class estimates 'in advance' of an event based on previous experience (e.g. each person says 'I think it will grow about ? cm this week') and compare the actual growth with the estimates.

B6 investigate relationships between measures for different attributes and apply to solve problems

Possible activities:

- Investigate relationships between perimeter and area (e.g. sketch and interpret tables and graphs showing the areas of different rectangles with the same perimeter and the perimeters of different rectangles of the same area).
- Investigate situations involving surface area and volume (e.g. take a sheet of paper and investigate to find the largest open box you can; display graphically the results of your investigation.)
- Use the notion of scale in maps and drawings (see **Space**).
- Investigate the effect on perimeter, area and volume of enlargement in scale of simple two- and three-dimensional shapes (e.g. those made up of squares and cubes respectively).
- Devise rules for finding the perimeter and area of a rectangle given its dimensions, and use the rules to solve practical problems where certain dimensions are unavailable for direct measurement or where direct measurement is impractical.
- Investigate the relationship between the diameter of a circle and its circumference (e.g. sketch and interpret table or graph of the circumference of circular lids of different diameters). No formulae will be necessary at this stage.
- Investigate the effect of change of shape on the mass, volume and surface area of a piece of play dough or clay.
- Sketch informal graphs to model events familiar to students, such as variations in hunger through the day.

MEASUREMENT in BAND C

Overview

During the secondary years, students should improve their measurement skills and their ability to estimate quantities. They should become proficient with commonly used measuring equipment, develop a good feel for the size of various standard units and be competent at estimating in standard units.

Students should recognize that all measurement is approximate and that efficient measurement requires a sensible choice of unit. To measure a room in centimetres and perform an accurate computation of the wall area may be impractical and inefficient if the decision is between buying one-litre or four-litre cans of paint and two one-litre cans cost about as much as a four-litre can. In some situations, the most accurate unit possible may be chosen; in other situations a rough measure may be the best choice.

A range of sensible methods of indirect measurement (mensuration formulae, Pythagoras' theorem, rates and differences, similarity and scale) should also be developed during the lower secondary years. The development of these concepts needs to be synchronised with the development of the concepts of common fractions and ratio (**Number**) and scale (**Space**). Techniques should be developed from, and applied to, a wide range of contexts to suit the variety of experiences of students. Some of the more common measurement formulae, such as those for plane figures, could be memorised, although the more complex formulae (e.g. those involving cones) may be looked up as needed.

Students should carry out practical tasks involving measurement; they must plan, make judgements about which measurements to make, organise and carry out the measurements, and decide whether the results are of the right order. Investigations should be realistic and should require that students, individually and collaboratively, choose and use the appropriate skills. These kinds of experiences are described in more depth in **Choosing and using mathematics**.

Students should learn to rearrange formulae (variously called 'transposition' or 'changing the subject'). While substitution is an alternative strategy to transposition, the latter skill is widely used in the workplace and therefore students need access to it. While students may learn to transpose as an algebraic manipulative technique, the process is likely to make more sense when applied to formulae where the letters have some intrinsic meaning. Consequently, transposition of formulae should precede the equivalent manipulation of abstract algebraic expressions, indeed, the latter may well build upon the former.

An important idea which should develop from the investigation of right triangles is that similar right triangles have equal corresponding ratios. This is the basic principle of right triangle trigonometry. The definition of the trigonometric ratios in right triangles have, therefore, been described as *possible* experiences in the list of points provided under the scope statement relating to similarity. Students may use the trigonometric function keys on their calculators to explore the values of the ratios in right triangles for specific angles.

The more formal treatment of trigonometric ratios may be delayed until unit circle and right triangle approaches can be developed together. The time needed to address some of the newer areas of emphases suggested in this statement (e.g. **Chance and data**, **Choosing and using mathematics**) suggests that right triangle trigonometry may not be a priority in band C. For those students for whom trigonometry seems appropriate during the lower secondary years of schooling, activities may be drawn from band D.

MEASUREMENT AND ESTIMATION

Experiences in measurement and estimation should be provided which enable students to:

C1 understand the use of metric prefixes to unify measurements in a coherent way and use and design a variety of scales

Possible activities:

- Use standard metric prefixes and conversions such as 2536 mm = 253.6 cm = 2.536 m appropriately in the context of practical problems.
- Use basic rules of thumb for equivalences between metric units and commonly used imperial units (e.g. a metre is a bit more than 3 feet).
- Design, make, calibrate and test simple measuring devices (e.g. for time, for weighing).
- Directly measure quantities of length, capacity, mass, weight, time and angle.
- Investigate measurement systems past and present (e.g. investigate the use of the cubit in Biblical literature; the origins of other units of length, area or volume; metric equivalences of traditional units).
- Investigate the development of the measurement of time using the sun's shadows (understanding seasonal variations in shadows), hourglasses, candle clocks, sand and water clocks, calendars, pendulums (measuring time of swing), chronometers and analog and digital clocks.
- Investigate 'standard' capacities in common usage which are not SI units (e.g. 600 mL milk bottles, 750 mL soft-drink bottles, 'yards' in gravel trucks).

C2 choose and use appropriate techniques and tools to measure length, capacity and volume, mass, area, time and angle

Possible activities:

- Use standard units of length, mass, area, capacity and volume, time, angle and weight in a wide range of practical situations.
- Compare different units for measuring distance (e.g. nautical mile, kilometre, light year).
- Explain the relationship between length units (e.g. cm) and units for area (cm^2) and volume (cm^3) and explore dimension analysis in an elementary way.
- Use standard measuring equipment for finding length, mass, capacity, time and angle:

    length (ruler, tape measure, trundle wheel in millimetres, centimetres and metres);
    capacity and volume (measuring jugs calibrated in millilitres and litres);
    mass (balance beam, scales, calibrated in 10 g, 100 g and 1 kg);
    time (digital and analog clock);
    angle (protractor, Rotagram™).
- Read a variety of instrument scales on standard measuring equipment with linear and non-linear scales (meters and dials).
- Discuss the limits of accuracy of measuring instruments, and methods for extending these limits (e.g. to find the volume of a drop of water, collect 100 drops and find the volume).

C3 estimate length, capacity and volume, mass, area, time and angle and make statements about the degree of confidence associated with the estimate

Possible activities :

- Identify and measure everyday items to use as reference points for estimation (e.g. a one-litre carton of milk; A4 sheet of paper is 600 cm^2).
- Identify and discuss reference points for quantities not easily measured (e.g. the mass of a small car is about 1 tonne).
- Use everyday objects or events as referents to estimate physical quantities (e.g. that is about half the size of a milk carton so it is about 500 mL).
- Explore common rule-of-thumb methods used by people to estimate quantities.
- Use a square grid to estimate areas of any shape.
- Estimate the scale factor between an original object and an image of it.
- Distinguish situations in which under-estimates or over-estimates are preferable.
- Decide on upper and lower bounds for measures.
- Judge the range of acceptable error for given situations and purposes.

C4 investigate relationships between length, angle, area and volume

Possible activities:

- Maximise areas for given perimeter.
- Design optimal shapes given constraints (e.g. design a container from a rectangular sheet of paper to maximise the capacity of the container).
- Use a computer program to solve optimisation problems (e.g. find what size sector to cut out of a circle to form the cone with the largest volume).
- Investigate the effect on the surface area, volume and mass of changes in scale (enlargement or reduction).

INDIRECT MEASUREMENT

Experiences in indirect measurement should be provided which enable students to:

C5 make estimates of quantities which cannot be measured directly or conveniently

Possible activities:

- Make order of magnitude estimates of quantities not readily obtained (e.g. Fermi problems such as estimating Australia's daily or yearly consumption of alcohol or meat pies; number of babies born in Brisbane each day).
- Estimate in situations where it is inconvenient to measure more accurately (e.g. estimate how much money would be collected in a money trail).

C6 use measurement formulae to find perimeters, areas and volumes of common two-dimensional and three-dimensional shapes

Possible activities:

- Investigate and use perimeter and area formulae (not involving trigonometry) for rectangles, triangles, parallelograms and circles.
- Investigate and use surface area and volume formulae for spheres, cones, prisms and pyramids.
- Find areas and volumes of composite plane figures and solids.
- Distinguish between formulae for perimeters, areas and volumes by considering dimensions.
- Substitute values into formulae.
- Report on mensuration formulae developed by the Babylonians (ancient Egyptians, Greeks or Indians, ...).
- Undertake a historical investigation of attempts to calculate π and experiment to produce estimates of π.

C7 understand and use the derived measures of density, speed and other rates

Possible activities:

- Use the concept of rate in routine ways (e.g. apply rates to dressmaking, shopping and taking medicine).
- Determine rates such as heartbeats per minute, cents per kilowatt hour, kilometres per hour, dollars per kilogram, and use these rates to calculate component values.
- Plot data collected to estimate rates (e.g. estimate heartbeats per minute by fitting a line to data collected over various time intervals.)
- Convert from one rate to another (e.g. from kilometres per hour to metres per second).
- Use currency exchange rates (e.g. have students bring notes and coins from country of origin, compare and make imaginary exchanges).
- Compare the mass of various objects of the same volume, and the volume of various objects of the same mass, in order to develop the concept of density as the rate which indicates the mass per unit volume of an object.
- Use density in order to determine measures which are inaccessible (e.g. using an estimate of density of the human body, predict own volume from mass).
- Apply density as an indirect measure in a range of practical contexts (e.g. mass of an iceberg; how large is a kilogram of gold?).

- Compare distances moved in fixed times (e.g. distance run by members of the class in two minutes) and also the times taken to move fixed distances (e.g. times for people in a swimming race) in order to develop the concept of speed as the rate which indicates distance travelled per unit of time.
- Analyse trends in data as examples of rate of change (e.g. graph the winning Olympic times for the 100 m swimming or running events for women and men since the Games started — discuss reasons for some of the trends and speculate on future times.)

C8 use transposition to change the subject of a formula for use in practical settings

Possible activities:

- Use backtracking to rearrange the terms of a formula.
- Rearrange the terms in a measurement formula in order to facilitate the development of an appropriate calculator key sequence to complete an exercise (e.g. the area of a square is 5 square metres, what is the length of its side?).
- Compare the efficiency of using a formula with or without transposition (e.g. given the problem 'find the annual percentage rate of interest that gives a threefold growth after 10 years', compare calculator strategies based on a formula in different forms).

C9 interpret and apply information involving time and elapsed time

Possible activities:

- Use conventions related to time, such as time zones and daylight saving (e.g. find two cities in Australia on the same time as Kalgoorlie) and solve related problems (e.g. what is the latest time a plane must leave Melbourne to get to its destination on time?).
- Read timetables (e.g. for bus) and determine estimated elapsed time from them (e.g. use the bus timetable to decide how long a bus trip will take, or how long you will have for shopping).
- Compare different calendars in current use.
- Use timelines which show, for example, mathematical history, social history, art, literature or geology, to investigate time spans between significant events and to recognise different events occurring during the same era.
- Perform calculations involving elapsed time (or duration).

C10 use similarity and Pythagoras' theorem for indirect measurement in two- and three-dimensions

Possible activities:

- Produce scale drawings in order to estimate inaccessible lengths/distances and unknown angles (including triangulation).
- Construct scale models of real objects from measurements made (e.g. for scale models of local buildings) or from reference sources (e.g. for a scale model of the solar system).
- Estimate relative area of regions from scaled maps (e.g. estimate the relative area of Western Australia and Victoria from a map of Australia; estimate the relative area of China and Australia from a globe).
- Understand the distorting effect of different projections from three to two dimensions (e.g. compare apparent sizes of regions estimated from a Mercator projection map and a globe).

- Investigate right triangles and determine the relationship between the lengths of the sides.
- Use Pythagoras' theorem to establish that a triangle has a right angle and to determine unknown distances.
- Represent similar right triangles on graph paper, measure corresponding sides and calculate in order to establish that corresponding ratios are equal. Relate these ratios to the trigonometric keys on their calculator.
- Use the trigonometric ratios of sine, cosine and tangent in order to calculate inaccessible lengths and unknown angles.
- Using a calculator, investigate open-ended problems involving trigonometric ratios (e.g. in a triangle with three angles A, B and C, sin A + sin B + sin C = 2: investigate).

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MEASUREMENT in BAND D

Overview

Basic measurement and estimation concepts and skills will have been established through the activities drawn from bands A to C. In band D, however, all students should be expected to increase their facility in choosing, making and interpreting measurements. They should:

- choose and make appropriate measurements;
- provide measurements in a suitable form;
- know the effect of errors when combining measures (e.g. adding them);
- give reasonable explanations of their methods and results.

These skills may be developed through the study of particular areas of application (e.g. health, surveying, meteorology, sport, horticulture) or in the context of modelling problems (e.g. related to design work, economics, mechanics, stock control, or the environment). In either case, students should engage in practically based activities which may range from designing, costing and marking out a garden sprinkler system, to determining the best shape for a carton of 750 mL cans, or making a study of the nautilus shell. Clearly, there is a close link between this and other strands, in particular, **Chance and data** and **Choosing and using mathematics**.

During bands A to C, students will have developed the notion that any measurement on a continuous scale involves error; in band D a more formal study of error (or tolerance) will be appropriate for many students. Calculators make the study of error arithmetic practical for all students. They should understand the importance of relative error and investigate the effect of errors when measurements are combined, for example, in finding volumes or using a conversion formula.

Many of the measurements students will need to obtain will require the use of indirect measurement techniques. Activities in band D should lead to an expansion of these techniques and include rates, circular measure, and the use of similarity relationships, including the trigonometric ratios, in two and three dimensions. Students should link their use of similarity in measurement to their work with the enlargement transformation and with scalar multiplication of vectors (**Space**, D2). They should also see triangle trigonometry as an application of more general principles about similarity. The application of trigonometry to situations involving triangles is focused upon with this strand but the connections between the trigonometric ratios and the circular functions (see **Algebra**, D3) should be studied quite explicitly. Historical studies in mathematics may be one useful technique for helping students integrate these ideas.

The subsection *Approximation, change and the calculus* first appears in band D. All students should develop some of the basic notions of functional relationships between measurable quantities, and of the rate of change of a function with respect to its variable. By finding average rates of change of some quantity (e.g. temperature) over progressively smaller time intervals, students can get a clearer picture of the change in the quantity over time. By graphing this quantity against the variable time, they can relate the computed 'average rates of change' within each time interval to the shape of the curve in that interval. The necessary calculations to undertake such tasks can quickly be carried out with a calculator.

These activities should lead to the related ideas of instantaneous rate of change and the gradient of a curve at a point. These may involve investigations such as that of the cooling of body temperature after exercise (of interest because too rapid a cooling may cause muscle damage). Using an algebraic model of body temperature after exercise, and the zoom facility of a graphics package, students can estimate the instantaneous rate of change at any particular time.

Students may then use a calculator to compute the average rate of change over small intervals and plot these values against time. Computer software is also available which will plot the instantaneous rate of change of a function at any point and hence produce a graph.

Graphing the instantaneous rate of change itself as a function of time leads to the idea of the derived (or gradient) function; that is, the function whose value at time t is the instantaneous rate of change of the original quantity (say, temperature) at that time. Comparison of the graph of the original function with that of the derived function helps students to understand the idea of the derivative and to conjecture about the derivative of particular functions. For example, students may make a direct comparison of the graph of the cosine function with the graph of the derived function and conjecture about the general expression for the derivative function.

The discussion of the formal definition of derivative as a limit can be assisted by the use of well-chosen examples. These could be selected from the physical, social and biological sciences, where there is a plausible case for accepting the idea of an instantaneous rate of change of one measurable quantity with respect to another.

All students should also consider the task of approximating areas and volumes of irregular shapes and develop some strategies for doing so. These strategies could range from finding volume by water displacement, to 'rules of thumb' for finding the volume of foundation needed for a house, and the use of techniques of calculus. Computer software is now available which makes accessible to many students the idea of approximating the area under a curve by partitioning it into rectangles. Through such techniques, they may use continuous functions to model a variety of situations.

The mathematical notions developed from these activities are those of a limiting value of an expression or an infinite sum and are related to ideas explored in the study of sequences and series (**Number**, D3, D4). They are recognised as involving concepts which are difficult to formalise. The levels to which formal and abstract presentations of these ideas are expected will vary according to the extent to which they are needed. For some students, an appropriate treatment will include a discussion and presentation of the rules for differentiating sums, products, etc., and a similar approach to the properties of integrals.

The types of functions needed to model situations are likely to include various algebraic and transcendental functions and their combinations. Some manipulative facility is required in order to use these mathematical models with confidence and to obtain correct results. The development of manipulative skills divorced from any context of their use is, however, inappropriate.

MEASUREMENT AND ESTIMATION

Experiences in measurement and estimation should be provided which enable students to:

D1 choose appropriate techniques and tools to measure quantities to chosen or specified levels of accuracy

Possible activities:

- Use a scientific calculator in different modes — degree and radian — to calculate measures relating to angles.
- Explain why and when rules of thumb can be used (e.g. halving °F for cooking in °C but not for daily temperatures).
- Plan and execute measurements appropriate for a specified task (e.g. take measurements which are adequate to enable the purchase of sheet insulation for walls).
- Analyse the possible effect of errors on calculations involving measurements (e.g. if a certain quantity has a mass of 2 kg, what is the effect on the error of measurement of calculating one hundred times the mass? … of calculating one hundredth of the mass?).
- Formalise rules for combining errors (e.g. use a general rule to calculate the range within which the area of a square lies, knowing the bounds for the length of its side).

INDIRECT MEASUREMENT

Experiences in indirect measurement should be provided which enable students to :

D2 use rates to deal with practical measurement tasks

Possible activities:

- Analyse various rates (e.g. heart rates, birth rates, unemployment rate, scoring rates in sports, speeds), graph relationships between relevant variables and discuss how rates may be used to make comparisons in practical situations.
- Discuss and explain statements about rates (e.g. as mortality rose, fertility rose) in terms of the technical meaning of a rate, distinguishing states (e.g. number of births) from rates (e.g. rate of births per live population).
- Apply fixed rates (e.g. unit costs, postage rate, taxi fares, agents' commissions) to problems.
- Relate steady rates to the slope of the graphs of linear functions (e.g. constant speed, straight line depreciation).
- Interpret changes in slope in the context from which they arise (e.g. changes in slope of a non-linear function, examination of relative maxima and minima and the slope of exponential curves).

D3 apply similarity relationships involving length, area and volume

Possible activities:

- Investigate realistic scaling situations which can be dealt with using mathematical models (e.g. use a rectangular prism or cylinder to model human trunk and legs in order to investigate why people who grow excessively will often be susceptible to broken legs).
- Use similarity arguments to obtain measurement formulae (e.g. sine rule, area of triangle) for triangles.
- Use relationships involving radius and angle subtended at the centre of a circle to find arc length, and area of a sector and segment.
- Apply circle formulae involving earth and planetary measure (e.g. shortest paths between two cities, distances on latitudes).
- Use appropriate tools for indirect measurement and investigate the underlying theory (e.g. thumbstick, stadioscope, clinometer, hypsometer, theodolite).

D4 use trigonometric ratios to solve problems in two and three dimensions

Possible activities:

- Apply rules relating to triangles in a range of practical contexts in two or three dimensions (e.g. sine rule, cosine rule, area formulae, applied to surveying, navigation).
- Investigate special cases of the cosine rule and area formulae (e.g. when the included angle is a right angle).
- Research and report on the history of trigonometry including the types of questions that led to its development and later developments leading to new applications, and link the triangle and circular approaches to trigonometry.

APPROXIMATION, CHANGE AND THE CALCULUS

Experiences in approximation and change should be provided which enable students to

D5 investigate and use instantaneous rate of change of a variable when interpreting models of situations

Possible activities:

- Graph data in which there is a variable rate of change of physical phenomena (e.g. body temperature, speed, and flow rates from non-uniform containers) and describe the rate of change informally (e.g. this graph shows that her body cooled faster initially and then the rate of cooling slowed down).
- Use actual data sets and investigate average rate of change over different intervals (e.g. investigate height of water in a non-uniform container by calculating the average rate of change over different time intervals and compare results with the graph of the relationship). Note: Computer software is available which simulates many such situations.
- Use the zoom facility on a graphics package to magnify progressively smaller portions of a graph to approximate instantaneous rate of change.
- Graph (by hand or using suitable software) the instantaneous rate of change of a quantity as a function of the variable leading to the idea of the derived (or gradient) function.
- Investigate and informally establish the derivatives of common functions (e.g. polynomials, trigonometric, exponential, logarithmic, rational).
- Use the calculus to find maximum and minimum values of functions, both as an aid to curve sketching and in order to develop a better understanding of the relationship between functions and their derivatives.
- Informally establish differentiation algorithms (e.g. products and quotients of functions).
- Find the 'best' way of doing something (e.g. quickest or cheapest, uses the least amount of materials, or makes the the biggest profit) by making assumptions, setting up functional relationships between two variables, and using differentiation in order to find maximum or minimum values.
- Use composite functions to describe rate problems which are a function of two variables (e.g. the pressure experienced by a hang glider or a balloonist is a function of height/time).
- Use anti-derivatives to establish a function given its gradient function or derivative (e.g. find the size of a population at a specific time given its rate of change and other known conditions).

D6 use suitable methods of approximation to find areas and volumes of irregular shapes

Possible activities:

- Develop and compare a range of strategies for estimating the area of irregular shapes and related quantities (e.g. approximating to common shapes, counting squares, random number techniques, Maples' method).
- Use a range of strategies (e.g. filling, slicing, weighing and displacement) to estimate the volume of irregular solids and related quantities.
- Investigate the history of attempts to find the area and/or volume of mathematical shapes (e.g. Galileo's efforts to find the area under a cycloid).

- Estimate areas of irregular shapes by covering them with square grids and progressively reducing the mesh in order to obtain a better estimate for the total area.
- Investigate methods of improving estimates for the volume of irregular shapes.
- Find the accumulated amount of some quantity by summing over the columns of a histogram (e.g. use a histogram which shows number of people in each age group to find the total number between specified ages).
- Relate the area under a curve to the accumulated amount of some quantity within a specified domain (e.g. given emissions of a particular gas from a factory as a function of time of day, connect the accumulated emissions during a day to the area under the curve of the graph of the function).
- Given a quantity as a linear function of time, determine the accumulated amount of the quantity within a specified time arithmetically and geometrically (e.g. emission of the gas increases steadily through the day: determine the total emission for a day arithmetically from the algebraic expression, and geometrically by finding the area under the curve).
- Given a quantity as a non-linear function of time, estimate the accumulated amount of the quantity within a specified time by slicing the area under the curve into rectangles.
- Use computer routines to estimate area under a curve (e.g. mid-ordinate and trapezoidal rules).

D7 investigate the use of infinite and limiting processes in integration

Possible activities:

- For a simple linear function (e.g. $y = 2x$) estimate the area under the curve within a specified domain by sectioning into rectangles, and progressively reducing the width of the rectangles to improve the estimates.
- For simple non-linear functions (e.g. $y = x^2$) estimate the area under the curve within a specified domain by sectioning into rectangles, and progressively reducing the width of the rectangles to improve the estimates.
- Write a general expression for the estimate of the area under the curve for a simple non-linear function (e.g. $y = x^2$) based on rectangles, and use a calculator or computer routine to find estimates of the area under a curve as the widths of the rectangles get smaller.
- Use a limit argument to determine the area under a curve (i.e. definite integral) within a specified domain.
- Investigate the integrals of common functions (using software where appropriate).
- Compare the results of calculations using integration methods with those obtained by methods of approximation.
- Informally establish simple integration algorithms (e.g. area bounded by two functions, integration by parts).
- Verify integrals in standard integral tables by differentiation.
- Use a calculator or computer to estimate and graph the indefinite integral of simple functions (e.g. sin x, $^1/_2x$) over a domain sufficient to enable students to guess the corresponding function and compare by graphing their guess.
- Consider graphically the indefinite integral of simple functions and define a primitive function (leading to Fundamental Theorem of the Calculus).

- Relate indefinite and definite integrals and use primitive functions to evaluate definite integrals.
- Interpret an integral in terms of the real phenomenon it represents. For example, the velocity in metres at which a monorail slows in case of a fault is given as a function of time t in seconds by: $\frac{100}{1 + 100kt}$, $k>0$

  What information is given by:

  $$\int_{t=0}^{60} \frac{100}{1 + 100kt}\, dt$$
- Apply integration techniques to situations which are modelled by the rate of change of a variable or variables in order to find the magnitude of the variable at a given state or time.
- Use integration techniques to find areas and volumes of revolution.
- Use slicing techniques to find volumes of various types of solids.

Chance and data

Purpose and scope

A sound grasp of concepts in the areas of chance, data handling and statistical inference is critical for the levels of numeracy appropriate for informed participation in society today. 'Data' provide us with a powerful means of forming opinions and reaching conclusions quite different from those we would reach if we relied upon, for example, 'authority' or 'hearsay'. Assessing the credibility of arguments based on information given should be the basis of the study of chance and data.

As the amount and variety of quantitative information confronting people have increased, however, so too has the need to understand the strategies for data collection and analysis, the nature of chance processes and the effect of chance on data collection, and the assumptions that underlie procedures and predictions. Statistical inference underlies such diverse matters as weather prediction, economic indicators, medical and other research design, risk insurance, gambling and quality improvement. Ultimately, it affects the lives of all people individually and collectively.

The teaching of **Chance and data** should be integrated throughout the curriculum, and not be limited to mathematics lessons. This will be most easily achieved in bands A and B, and is highly desirable in the other bands as well. This strand is described under three subheadings: *Chance*; *Data handling*; and *Statistical inference*.

Chance

Many situations in the adult world involve chance. For example, gambling is a significant part of the lives and customs of many Australian adults, as are decisions about the insurance of health and property. Chance is also a familiar part of children's lives. They ask whether the new baby will be a girl or a boy or whether it is likely to rain on the day of the picnic, and they play games and enter competitions which involve chance. It is through such questions and activities, and before formal schooling, that children begin to develop their understanding of chance processes. The language of chance is widely used in a colloquial way (tomorrow will probably be fine, it's an odds-on bet, it's a sure thing). Students should be helped to refine and extend their use of this language so that they are more able to make sense of their everyday experiences. The concept of chance has wide applications throughout the school curriculum.

Misconceptions about chance processes are widespread. Many become established while children are still quite young and are then difficult to overcome. Therefore chance activities should be provided in schools from the earliest stages in order to help students develop more inclusive conceptions. Such activities also provide the basis for more formal study in later school years. Students should investigate chance events, estimate probabilities experimentally (that is, empirically) and determine them analytically (that is, theoretically). Many situations which are difficult to deal with analytically and which cannot be directly experimented with are readily accessible through simulation methods. Students can solve quite complex problems using simple simulation techniques, particularly if appropriate computing facilities are available.

Data handling

The achievement of confidence and competence to make sense of and interpret data which have been collected, organised, summarised and represented by others is a major goal for this strand. If they are not to be subjected to the kind of exploitation implied by the expression 'you can prove anything with statistics', students need experience in judging the quality and appropriateness of data collection and presentation for answering the questions at hand. They should interpret tables, charts and graphs which range from simple to complex, and from those which provide very good models of data presentation, through those which are flawed but still communicate in useful ways, to those that are misleading.

Most people need only occasionally to collect and handle raw statistical data and convey such data to others. Nonetheless, students should have considerable opportunity to collect, organise, represent and interpret data from situations of interest to themselves, in order to answer questions which they and others have posed. This should help them to see a range of purposes for collecting data and to develop an understanding of data collection and handling processes.

Technological changes have influenced the techniques of data collection, retrieval, manipulation, analysis and communication, and have increased our capacity to pursue investigations with large quantities of real and simulated numerical data. The school curriculum should reflect these changes and students should have the opportunity:

- to use computing technology to enable them to handle data drawn from everyday contexts;
- to engage in extended, practical investigations;
- to consider the social implications of the impact of technological change on the collection and handling of data.

Statistical inference

The dual notions of sampling and of making inferences about populations, based on samples, are fundamental to prediction and decision making in many aspects of life. Students will need a great many experiences to enable them to understand principles underlying sampling and statistical inference and the important distinctions between a population and a sample, a parameter and an estimate. Although this subheading first appears separately in band C, the groundwork should be laid in the early years of schooling in the context of data handling and chance activities.

The making of statistical inferences in such diverse areas as weather prediction, town planning, economic forecasting, scaling test results and stock control all require the construction of mathematical models to represent aspects of the 'real' situation in a simplified way. Students should understand that the models themselves are *not* the reality and that different models may be applied to the same phenomena. Different models will be based on different assumptions and may result in different predictions. Consequently, students should learn to question the assumptions underlying data collection, analysis and interpretation and the reasonableness of inferences and conclusions. (See **Choosing and using mathematics**.)

CHANCE AND DATA in BAND A

Overview

During the early primary years, children should carry out experiments which involve chance processes (e.g. toss a die) and examine the outcomes; that is, they should try an 'experiment' and note the 'results'. Any preliminary discussions and predictions should refer to the *range* of possibilities — not to the probable occurrence of these — and even the range of possibilities need not be formalised.

Young children should have considerable opportunity to clarify the use of language related to chance events. For example, they should use such words as 'likely', 'certain', 'probable' and 'impossible' appropriately in context. Discussion of their experiments and chance aspects of their daily lives should be based on natural language use and ideas should be expressed in the children's own terms. While the teacher's use of correct (but not technical) language is important, the use of the formal or technical language of chance is not expected.

During these early years, children should also begin to develop the notions that some form of classification underlies the collection and organisation of data, and that how one classifies depends on the purpose of the classification. For example, collecting information on 'the most popular kinds of toys' or on 'the most hard-wearing kinds of toys' might sensibly require different classifications of toys. The emphasis throughout should be on recording and representing data which arise from the practical and problem-solving activities in which the children are engaged. Opportunities occur in work from all of the mathematics strands and across all curriculum areas.

Initially, there should be an emphasis on the use of the actual objects or physical representations of objects or measurements, although children should gradually move to simple pictorial, block and bar graphs. Thus students may construct concrete or pictorial graphs to compare collections by one-to-one correspondence (e.g. to compare how many children have short or long sleeves, two lines of children can act as a concrete graph, while a pictorial record of the same makes a pictorial graph, and tokens for each child can be used to make a block graph). The lengths of various body parts can be similarly compared in a variety of representations. Students might also keep daily records of the growth of a sweet potato plant, or record the number of teeth lost in the class for each month of the year and decide upon strategies for recording and presenting the information.

Some of the data that children deal with will be discrete (e.g. the number of teeth lost each month) and other data will be continuous (e.g. the length of the sweet potato plant) and these require different data-handling techniques. The appropriateness of the data collection and representation strategies for different types of data may be considered during these early years but the words 'discrete' and 'continuous' are not appropriate. Whether the 'graphs' are concrete, pictorial or more symbolic, the need to use a common baseline when comparing frequencies or measures is a fundamental notion which is not at all obvious to young children, and which develops gradually as they gain experience in using graphs to compare occurrences.

CHANCE

Experiences with chance should be provided which enable children to:

A1 use, with clarity, everyday language associated with chance events

Possible activities:

- Clarify and use common expressions such as 'being lucky', 'that's not fair', 'always', 'it might happen', 'tomorrow it will probably rain'.
- Use the vocabulary 'certain', 'uncertain', 'possible' and 'impossible' appropriately, recognising that, while there is an element of uncertainty about some events, others are either certain or impossible (e.g. I think it is certain that our teacher would be over seventeen because you usually only finish high school then).
- Use language such as 'very likely', 'unlikely', 'more likely', 'less likely' and 'equally likely' to describe events which relate to the experience of the child (e.g. how likely are the events 'we will do some maths in school today', 'the egg will crack if I drop it'? is it more likely that it will rain in July or that it will rain in January?).

A2 describe possible outcomes for familiar random events and one-stage experiments

Possible activities:

- Identify all outcomes arising from one-stage chance experiments, such as tossing a coin, rolling a die, using a spinner, or selecting a marble from a container.
- Record outcomes from simple experiments involving counters (e.g. toss 8 counters and record the numbers falling each side of a piece of cotton, saying, for example, that 'three fell one side and five the other side and none on the cotton, and three and five make eight').
- List possible outcomes from one-stage experiments involving counters (e.g. select a fixed number of counters, say six, which have different coloured faces and list all the possible outcomes when the counters are tossed together).
- Use techniques for random assignment of roles (e.g. coin tossing, spin the bottle, and children's rhymes, where often the only random part is the starting point).

A3 place outcomes for familiar events and one-stage experiments in order from those least likely to happen to those most likely to happen

Possible activities:

- Select from containers with different numbers of each of two colours of Smarties according to how likely it is that a particular event will occur (e.g. from which container am I more likely to pick out a red than blue?).
- Place familiar (imagined or real) situations in order from the one believed most likely to the one believed least likely (e.g. tomorrow will be Sunday; the glass will break if you drop it; I will toss the coin and get a head; the Principal will come to school on Thursday with green hair).
- Investigate such questions as 'is it really harder to throw a six than any of the other numbers or does it just seem that way sometimes?' and discuss in terms of equally likely events.

DATA HANDLING

Experiences with data handling should be provided which enable children to:

A4 frame questions about themselves, families and friends, and collect, sort and organise information in order to answer these questions

Possible activities:

- Devise rules to sort and classify objects and use the rules consistently (e.g. classify the things in their pencil cases).
- Make decisions about what data to collect in order to answer questions about situations familiar to children (e.g. what pets are the most popular? how many leaves do the snails eat each day?).
- Organise persons and objects (toy animals, building blocks, counters, pictures) into 1–1 correspondence in order to answer comparison questions (e.g. do more children like tacos or do more children like lasagna?).
- Make tallies of data to collect frequency information (e.g. the number and type of animals seen during the day).
- Decide how to collect measurements in order to answer simple questions about themselves (e.g. how do we decide who has the longest arm in the class?).
- Organise data in order to answer questions about themselves (e.g. organise data in order to decide who has the longest arm and whether the same person is tallest).

A5 represent and interpret information to answer questions about themselves, friends and families

Possible activities:

- Represent persons and objects using concrete representations, or pictorial versions such as Venn diagrams, arrow diagrams and simple trees.
- Represent collected discrete data using objects and drawings (e.g. a picture of a taco for each student who chose tacos as 'favourite') in order to make comparisons (e.g. a pictorial block graph).
- Represent collected continuous data using paper strips (e.g. a strip of paper as long as the distance around the head) in order to make comparisons (who has the biggest head?).
- Represent familiar situations (e.g. use paper strips to represent class members' height) and investigate the need for a baseline for comparisons to be made.
- Extract specific pieces of information from tables, lists and other simple representations of data (e.g. read a value from a table).

CHANCE AND DATA in BAND B

Overview

During band B, children should continue to collect, present and interpret data in order to answer questions of interest to themselves and to further develop their understanding of chance processes.

In investigating chance, children should order familiar events informally from most likely to least likely. They should also make complete lists of possible outcomes from simple 'experiments' (e.g. tossing a die) and demonstrate how each outcome may occur. The ability to predict outcomes in order from most likely to least likely should develop during this period and some children *may* begin to assign numerical probabilities to events which are simple to analyse (e.g. die throwing, coin tossing, drawing cards from packs). While it is important that this does not assume an understanding of fractions (**Number**) which children do not have, work in chance may provide one of the contexts for the development of children's concepts of fractions.

Children should experiment with, discuss and compare the results of chance processes and note that separate experiments will usually produce somewhat different results. An important understanding to develop is that while the results of a single trial cannot be predicted, the results of a large set of trials can be predicted to some extent.

They should also compare situations where a census is needed with situations where a sample may be appropriate and begin to carry out simple sampling activities generated from a range of cross-curriculum experiences. In conducting simple surveys, the processes involved in gathering the information are as important as the results obtained. Children should decide:

- the type of information that is to be collected, and the people (things) to be surveyed;
- how to collect, record and organise the information;
- an appropriate way to display the information to others: for example, a table, diagram or other representation.

Children should learn that data can be displayed in a variety of ways and that the choice of display depends upon the question being asked of the data. Graphs should not be regarded as an end in themselves; rather they should serve purposes which are clear to children. As the children perceive the need for increasingly sophisticated forms of data representation, the teacher can assist them by introducing new methods of representation. Little is likely to be achieved by providing a collection of data (found in a text) and having children practise drawing graph types in isolation. However, children should begin to locate and interpret data from a variety of sources, including encyclopaedias, textbooks, newspapers and magazines, and to incorporate the results of their research into reports of their own.

During these years, children will begin to make informal inferences on the basis of samples and discuss whether it is possible to generalise. For example, they may decide to use the head sizes of all those in Year 6 at their school to make predictions about head sizes for other 11-year-old children, but decide not to generalise to much younger or older people. They could ask whether their school, or their class, is an appropriate sample from which to draw conclusions about a larger population. They should be encouraged to question the appropriateness of generalising beyond the bounds of the data collected and begin to consider the circumstances under which generalisations may reasonably be made.

CHANCE

Experiences with chance should be provided which enable children to:

B1 make statements about how likely are everyday experiences which involve some elements of chance and understand the terms 'chance' and 'probability' in common usage

Possible activities:

- Make simple predictive statements about everyday events (e.g. in Perth, it is more likely to rain in July than in December) and understand and use appropriate words such as possible/impossible, likely/unlikely, certain/uncertain, biased/fair, a chance/no chance.
- Discuss situations in which one may wish to maximise or minimise the possibility of certain events occurring (e.g. there is usually less traffic on the road I take to school than on the other road; I am less likely to have an accident than if I took the busy route).
- Interpret statements of numerical probability in common use (i.e. use the idea that very likely events have probabilities 'close to 1' and very unlikely events have probabilities 'close to 0' to interpret probability statements).
- Interpret alternative ways of expressing probabilities which are in everyday use (e.g. the probability of rain tomorrow is 30 per cent; there's a fifty-fifty chance I will go to the concert).

B2 for random events, systematically list possible outcomes, deduce the order of probability of outcomes and test predictions experimentally

Possible activities:

- Systematically list all the ways an event can occur (e.g. list all the ways of scoring a sum of seven when throwing two dice).
- Make non-numerical predictions about equally likely events, such as those involved in rolling a fair die, and compare predictions with the results of experiments (e.g. you are more likely to get a five and a six than two sixes).
- For simple experiments with dice and spinners make non-numerical predictions (e.g. order from most likely to least likely) for events which are not equally likely (e.g. cubes with 1, 2, 3, 4, 5 and 5 written on the faces, or a coloured spinner on which the sectors are unequal) and compare predictions with results.
- Choose the experimental situation in which an event is more likely to occur (e.g. from which container are you most likely to draw a red ball?).
- Analyse simple experiments (e.g. those involving single coins, dice and simple spinners, and equally likely events); make a systematic list of possible outcomes and assign simple numerical probabilities based on reasoning about symmetry.

B3 make and interpret empirically based predictions about simple situations

Possible activities:

- Investigate situations which are difficult or impossible to analyse theoretically but experimentally simple (e.g. a drawing pin falling point up) and begin to develop the notion of the relative frequency of different outcomes for chance events.
- Make simple numerical statements about the probability of events based on past results.
- Conduct experiments with coins, dice and spinners; record and organise the data,

and compare the results with predictions.

- Seek from appropriate sources information concerning the risks involved in certain situations (e.g. the risk of accidents per 1000 km travelled for different modes of travel).
- Give and justify subjective estimates of probabilities in a range of familiar situations (e.g. I think that the chance of there being lots of traffic today is very small because usually ...).

DATA HANDLING

Experiences with data handling should be provided which enable children to:

B4 systematically collect, organise and record data to answer questions posed by themselves and others

Possible activities:

- From a simplified road map identify all possible routes between two places.
- Pose questions about situations which are familiar to them (e.g. what makes us feel happy? which foods from the school canteen are the most/least popular?).
- Prepare short questionnaires (two or three simple questions) to enable data collection, discussing the need for consistent presentation of questions and the possibility of different interpretations of questions.
- Design data collection sheets and use them to record and organise data (e.g. record the number or type of cars owned by students' families).

B5 represent, interpret and report on data in order to answer questions posed by themselves and others

Possible activities:

- Make short statements about data (e.g. more than half the class walked to school each day this week).
- Represent data in tables and graphs and compare different representations of the same data, considering how well they communicate the information (e.g. correct, clear, misleading).
- Discuss and interpret information presented in graphs and tables found in newspapers, magazines and text materials.
- Prepare and present oral or written reports to describe data collected, the source of the data, how the sample was chosen and how the measurements were made.
- Discuss and present conclusions drawn from data recorded elsewhere (e.g. movement in 'Top 40').

B6 understand what samples are, select appropriate samples from specified groups and draw informal inferences from data collected

Possible activities:

- Distinguish between populations and samples in situations where the distinction is clear.
- Discuss and decide how to select a simple random sample (e.g. by drawing names out of a hat, or by giving each child a number).
- Discuss when it is appropriate to use a sample rather than a census of all the population.
- Draw informal inferences based on samples (e.g. a 10-year-old child is likely to weigh about 40 kg).
- Discuss the reasonableness of drawing conclusions about populations from particular samples (e.g. whether it is reasonable to draw conclusions about the whole school from data based on our class).
- Discuss and make judgements about arguments and claims in the media for which statistical information is presented (e.g. claiming that 40% of the community think that the school leaving age should be raised on the basis of a telephone 'ring-in' poll).

CHANCE AND DATA in BAND C

Overview

The emphasis in **Chance and data** should continue to be practical and experimental rather than theoretical and formal. Students' use of language, however, should become more precise and subtle (although not technical) in reporting outcomes and communicating ideas. An appreciation of the social importance of chance, considering such matters as insurance, safeguarding the quality and reliability of products, opinion polls, election forecasting, and gambling, should also develop during these years. Students should be encouraged to draw upon their experiences in and make use of data gathered from such areas as geography, physical education, science, and environmental education.

Students should estimate probabilities experimentally and should develop an understanding of probability as the 'long-run relative frequency' (also called the 'long-run success fraction'). Simple sample spaces should be constructed and probabilities calculated from the analysis of situations. They should also set up simple chance models of situations and carry out simulations in order to estimate probabilities, means and medians (and possibly standard deviations), and to make predictions based on given probabilities (e.g. that 1 in 50 people will have an accident in a year). Simulations could be conducted using a computer only if students are already familiar with other forms of simulation (e.g. with cards or spinners).

Students should be encouraged to consider the problems associated with particular data sets and to research and extract information from various data sources in order to answer questions of immediate and future importance to them. They may, for example, use a variety of sources to investigate such matters as gender segregation of the labour market or youth unemployment.

Practical investigations should be undertaken which involve all of the stages of data handling (collecting, organising, summarising, representing and interpreting). Students may plan and execute surveys about student opinions on such matters as environmental and conservation issues or tastes in music, or collect census-type data from students, investigating, for example, the proportion of students who are bilingual. The activities should include careful consideration of:

- procedures for choosing samples and designing and trialling questionnaires;
- the comparative advantages of different methods of organising and representing data (including tables, graphs and summary statistics);
- difficulties which arise at the interpretation stage which can be used to inform later projects.

Many interpretations of data are based on summary statistics, such as measures of central tendency, variability and association, and graphs such as line plots, histograms, stem-and-leaf plots, box plots, scatter plots, and lines of best fit. Students should be able to interpret these various representations, understand the conditions under which their use is appropriate, and compare and select from different possible representations of the same data.

Some of the questions which students investigate will result in moderate to large data sets. It is often inappropriate and inefficient to deal with these manually, and computers or scientific calculators should then be used. It is appropriate to have students calculate summary statistics and draw graphs manually for some small data sets, in order to learn the techniques and so deal with situations where very little by way of data manipulation is required. For all other situations the emphasis should be on the use of calculators and computers so as to minimise the drudgery involved and enable students to concentrate on interpreting the data. Scientific calculators are an obvious tool for the calculation of summary statistics, but a computer will sometimes be more appropriate because of the graphics, editing and more extensive calculating capabilities.

Students should distinguish between a population and a sample and interpret the commonly used population parameters (e.g. mean, median, proportion). Finally, they should informally draw inferences from data collected by themselves and others, construct convincing arguments based on such data and evaluate arguments based on data.

The scope statements in the data-handling component of this strand suggest plots and measures which have become routine tools for statisticians but have not been common in schools. Some go beyond what has normally been regarded as appropriate for all students. However, the gradual conceptual development of these ideas from band A, and through band B, together with the use of calculator or computing technology, should enable all students to develop these important skills.

CHANCE

Experiences with chance should be provided which enable students to:

C1 understand and explain social uses of chance processes

Possible activities:

- Investigate uses of probability in insurance and games of chance, and discuss related social issues.
- Recognise that, in historical terms, the study of chance and randomness is relatively new but has had a profound effect on almost every aspect of our lives.
- Study common games of chance by analysis and simulation to find the probability of winning a major prize, any prize, the expected return on the dollar, etc.
- Discuss the social implications of public gambling (e.g. poker machines, two-up, Lotto).
- Describe probabilities on a scale between 0 and 1 and place informal expressions of chance on the scale (e.g. from impossible, through poor chance, even chance, good chance and certain).
- Use the language of chance to describe events which are mutually exclusive (e.g. if one of these does happen then the other will not), including the special case of complementary events.
- Discuss the term 'odds-on' and note that statements of odds which appear in gambling contexts reflect statements of subjective probability as well as statements of return on money invested (e.g. recognise that if the odds are 100 to 1 then the return on a win is very high but the chance of a win is correspondingly small).
- Investigate 'odds' to determine how the bookmaker makes a profit (the related probabilities add up to more than one).

C2 construct sample spaces to analyse and explain possible outcomes of simple experiments and calculate probabilities by analysis of equally likely events

Possible activities:

- Describe the sample space for simple one-stage events involving coins, dice, cards and spinners and drawing balls from containers (e.g. describe the sample space for 'getting an even number on the throw of a fair die').
- Describe the sample space for simple two- and three-stage events using such simple techniques as constructing organised lists and tables or tree diagrams (e.g. use a table to show the possible results of tossing two dice to investigate the event 'getting two as the difference of the two dice' or produce a tree diagram to show the possible results of drawing three balls from a bag).
- Solve simple probability problems, using the principle that, if an experiment has several possible outcomes which are exhaustive and mutually exclusive, the total of the probabilities of each is 1 (e.g. the probability of something happening is 1 minus the probability of it not happening).
- Determine the probability of simple events by reasoning about equally likely outcomes (e.g. upon drawing one card from a full pack the probability of getting a heart is one-quarter because …).

C3 estimate probabilities using the long-run relative frequency (that is, empirical probabilities)

Possible activities:

- Recognise that repetitions of the same experiment may produce different results.
- Devise and carry out simple experiments (e.g. dice, spinners, coins, tossing a thumbtack) to estimate probabilities; compare the results with results determined by analysis, and discuss the differences observed.
- Investigate the effect on the estimate of the probability as the amount of data collected increases (e.g. plot the proportion of successful trials against the number of trials and observe the convergence to the theoretical value).
- Recognise that while the analytic probability of, for example, tossing a 6 on a die may be one-sixth, this does not mean that one 6 will appear in every six tosses but rather that in the very long run 6 will appear roughly one-sixth of the time.

C4 model situations and devise and carry out simulations

Possible activities:

- Represent one chance event by another chance activity (e.g. use a spinner divided into seven sections to model the number of babies being born on any day of the week) and use simulations to estimate probabilities, means, medians, etc.
- Represent an event by a chance activity (e.g. given meteorological data about the empirical probability of rain in a particular month of the year use a spinner to simulate a sequence of rainy and dry days).
- Use random number tables, calculator random number generators or a computer when undertaking investigations involving simulation.
- Informally consider the number of simulations necessary to obtain a satisfactory result by noting how the 'estimate' obtained changes as the number of simulations increases.

DATA HANDLING

Experiences with data handling should be provided which enable students to:

C5 access, evaluate and interpret information presented in different forms from a variety of sources

Possible activities:

- Find sources of data, such as a yearbook or almanac, journals, magazines.
- Extract information from a variety of graphs, charts and tables presented in the media and government publications (e.g. Australian Bureau of Statistics Census Data, *Year Book Australia*).
- Interpret information presented in the media in terms of summary statistics (e.g. the median price of a house in Sydney is ...).
- Access and sort information in prepared computer databases.
- Critically comment on features of the data which might affect relevance and interpretation (e.g. are 5 successes in 10 trials as likely as 500 successes in 1000 to justify an estimate of 50% success?).

C6 systematically collect, organise and record data for practical purposes

Possible activities:

- Formulate key questions in order to investigate issues of interest to students (e.g. what would you need to ask in order to investigate the latest trends in fashion and music? what jobs are available for teenagers?).
- Design simple questionnaires and undertake trials to test whether the questions serve the intended purpose, and investigate the effects of different wordings of questions on responses.
- Structure and record data in a computer database so that it can be sorted in various ways to facilitate answering particular questions.
- Collect and organise time series data (e.g. traffic data, weather data).
- Design and conduct simple experiments to compare different treatments (e.g. the growth of seedlings and the amount of water provided each day).
- Look at the reliability of data (e.g. how accurately do students recognise and record the number of 'typos' on a page?).

C7 summarise and interpret data using visual representations and measures of location and spread

Possible activities:

- Distinguish informally between discrete and continuous data, and between categorical data (e.g. which country do you come from?) and ordinal data (e.g. which do you like most? ... least?).
- Develop techniques related to the production of simple or grouped frequency tables.
- Represent univariate data in standard graphical forms (e.g. line plots, histograms, stem-and-leaf plots and box plots), choosing forms, scales and axes which are appropriate for the context.
- Compare different representations of the same data, recognising the strengths and weaknesses of various forms of representation.

- Examine the appropriateness and effectiveness of various representations (tables, graphs and summary statistics) of information (e.g. consider the weaknesses of pie charts for representing small differences).
- Use and interpret measures of 'location' (also called 'central tendency') such as mean and median, compare the way they are affected by extreme values and recognise the median as the more robust measure.
- Discuss and investigate why mode is a poor measure of location.
- Use and interpret interquartile range and compare with range.
- Identify possible causes of 'outliers' in data sets, and investigate their effect on sample statistics (e.g. suggest what will happen to mean, median and mode of heights of students in a class when 'tall Wally' joins the class).
- Choose appropriate measures of location and spread for given circumstances.
- Given a scatter plot for bivariate data, indicate whether the association is weak or strong, positive or negative.
- Investigate and interpret relationships, distinguishing association from cause and effect.

C8 understand the impact of statistics on daily life.

Possible activities:

- Investigate the use of the word 'average' in a range of social contexts (e.g. consider the impressions given by 'the average student …' and 'the student with average results in mathematics …') and the use of different 'averages' to serve different purposes and sometimes to mislead.
- Investigate the role of statistics in shaping and describing aspects of public life (e.g. forecasting economic trends, describing and influencing consumer tastes, developing public perception and public policy on controversial issues).
- Explore some of the various forms of information collection and recording that have been and are being used (e.g. the tally stick, the *quipu* of the Mayans, church records, census, *Guinness Book of Records*, sport and weather information, computer networks, data banks and credit ratings).

STATISTICAL INFERENCE

Experiences with statistical inference should be provided which enable students to:

C9 understand what samples are and recognise the importance of random samples and sample size

Possible activities:

- Distinguish between a population and a sample and consider the circumstances under which a sample may or may not be appropriate or necessary.
- Take samples from populations in simple structured situations and use samples to make simple informal predictions about the population (e.g. sample from a bag of red and blue balls and make informal predictions such as 'I think there are about twice as many red as blue balls in the bag'). Check the predictions, discuss how good the predictions were and how they might be improved.
- Investigate the effect of sample size on predictions in simple situations (e.g. sampling from a bag of coloured balls to predict the composition of the bag, sampling from students in a class to predict the average height of students in the class).
- Carry out investigations where sampling is used to make a prediction (e.g. sample the students to predict how they will vote in the school elections).
- Discuss informally how samples can be unrepresentative.

C10 draw inferences and construct and evaluate arguments based on sample data

Possible activities:

- Develop inferences from data that may be in tables, computer databases, or scatter plots.
- Make predictions from lines of best fit.
- Draw inferences from time series data and predictions from trend lines.
- Investigate data in a computer database, analysing data from different viewpoints, drawing inferences and developing arguments.
- Evaluate information, by considering, for example, definitions of the terms used (e.g. how is 'unemployed' defined?), bias in questions, sampling procedures, design and conduct of an experiment.
- Use published information to assist in the development of informed opinions and arguments (e.g. about logging of forests in the national estate).
- Critically review surveys, polls and reports.

CHANCE AND DATA in BAND D

Overview

Chance and data should continue to be practical, with a major purpose being to assist students to:

- make sensible judgements about the quality of data which is to be the basis for decision making and about the confidence to be placed in inferences drawn from data;
- learn how to design simple surveys and experiments;
- recognise common statistical fallacies.

Underlying all the work in this strand is the development of the concept of a random variable. While a formal study of the concept is unnecessary, students should investigate a range of chance processes. They should identify random elements in everyday events and in data collection generally.

Students should use both theoretical and experimental approaches to probability, and understand the differences between them and the value of each. They should realise that probabilistic models, whether theoretical or experimental, make assumptions about the behaviour of phenomena. How well these probabilities predict 'real' behaviour is, therefore, essentially an empirical question. Students should investigate various probability distributions by experimentation, simulation or theoretical analysis.

In studying a number of frequency polygons formed from real samples, students should observe certain patterns which occur frequently. Students should make a special study of the functions which describe these patterns because they may be used to model a wide range of phenomena (see the link with 'functions' in **Algebra** and **Choosing and using mathematics**). They are usually called distributions or frequency curves and may include binomial, geometric, hypergeometric, Poisson, uniform, normal and exponential. Students should understand the assumptions which underlie the probability models they study and hence appreciate their limitations and the circumstances under which each may be used. They should distinguish between 'discrete' and 'continuous' models, and some may investigate relationships between models (e.g. how a binomial distribution can be approximated by a normal distribution).

Students should undertake statistical investigations to answer specific questions. This should involve them in all stages, namely, stages of design, conduct, analysis and report. The general processes for choosing, making and interpreting measurements described in band D of **Measurement** apply equally to the collection of statistical data.

Students should learn how to enter and access information in a simple database and appreciate the potential for errors in transcription and coding. They should explore raw data from a variety of sources, such as science, social science, history, politics, economics, public health, or market research.

Techniques of exploratory data analysis are employed by statisticians to provide an initial feeling for data prior to further analysis. While simplified versions of these techniques can be performed by hand, they are far more accessible when computers are used and software suitable for school use is now available. Starting with a set of data (e.g. from a survey, experiment, simulation or an established database), students should choose appropriate ways to classify and represent it, using visual representations, statistical measures (of location, spread and correlation) and curve fitting. They should discuss significant features of the data set, relating these features to the questions being investigated.

The essence of statistical inference is reaching conclusions about a population on the basis of sample data. Students should understand that populations may be finite and clearly defined (e.g. a school enrolment), or more difficult to define (e.g. the population of Melbourne), and that when we toss a coin we have a theoretical (infinite) population. Students should consider statistical, practical and economic reasons which may determine the choice of a sample or the design of an experiment, and the consequences of that choice for the confidence which can be placed on the conclusions.

All students should develop at least an informal understanding of confidence intervals, even if a more technical treatment of this is inappropriate. Confidence intervals can also be used as an informal approach to hypothesis testing.

Some students may undertake a more formal study of relationships between sample statistics and population parameters, testing hypotheses and calculating and interpreting confidence intervals. This study may also include the use of distribution-free statistical tests.

Although material in band D of this strand is grouped under three subheadings: *Chance, Data handling* and *Statistical inference*, there is considerable overlap among these. Some of the activities listed under each subheading require the use of concepts and/or techniques from other subheadings.

CHANCE

Experiences in chance should be provided which enable students to:

D1 use experimental and theoretical approaches to investigate situations involving chance and to determine the likelihood of particular outcomes

Possible activities:

- Use long-run relative frequencies to estimate probabilities from observed data (e.g. use meteorological data to estimate the probability of a sunny day in Perth in July).
- Plan and execute experiments to estimate probabilities from long-run relative frequencies.
- Investigate the behaviour of experimentally determined probability as the amount of data increases (e.g. plot the proportion of heads obtained against the number of tosses of a coin).
- Use a variety of representations of sample spaces (e.g. tree diagrams, Venn diagrams, lists) to calculate probabilities.
- Calculate the probability of events and compare with the result of an experiment (e.g. compare the theoretical and experimental results for a spinner with one sector red and the other blue).
- Illustrate and clarify the use of the technical language of probability (e.g. trial, outcome, sample space, event, random variable, disjoint, independent, mutually exclusive, conditional probability).
- Consider the social and ethical issues which relate to false negatives and false positives for screening tests and the balancing of costs and risks.
- Appraise instances of subjective probability (e.g. the odds given by a bookmaker or long-range weather forecasting).

D2 generate, use, interpret and investigate properties of probability models

Possible activities:

- Identify random elements in everyday events which involve chance.
- Investigate 'run lengths' for random processes using theory or simulations (e.g investigate the runs of sunny days in Perth in July based on a previously determined probability estimate and compare with actual data).
- Undertake practical activities (e.g. measuring heights, drawing cards, tossing dice, using a Binostat or a Geiger Counter) to generate and investigate probability distributions.
- Examine various probabilistic misconceptions (e.g. gamblers' run).
- Devise, play and analyse a variety of 'fair' and 'unfair' games.
- Examine various probability models (normal, binomial, geometric, Poisson etc.) and properties such as mean, quartiles, and variance of each.
- Calculate and interpret expected values for practical situations (e.g. return on lottery tickets, bookmaker's profit and expected life span).

D3 use simulations to model uncertain events

Possible activities:

- Carry out a simulation based on a probability model (e.g. estimate a value for π using Buffon's needle method, birthday or queuing problems).
- Use a variety of random number generators (e.g. dice, random number tables,

computer-generated 'random' numbers) in the course of simulations.

- Write a computer program to generate 'random' numbers and discuss issues surrounding computer-generated numbers.
- Compare theoretically determined probabilities with those obtained from a simulation.
- Make and justify decisions about the number of simulations needed in particular circumstances.

DATA HANDLING

Experiences in data handling should be provided which enable students to:

D4 plan, manage and appraise data collection and presentation

Possible activities:

- Prepare alternative forms of a survey questionnaire and compare responses to the two forms (e.g. a smoker and a non-smoker both design a questionnaire about smoking in public places).
- Investigate various influences on the quality of questionnaire data (e.g. non-response, respondents exaggerate or don't like the interviewer).
- Examine and compare different sampling methods (e.g. random, convenience, quota, systematic, stratified, cluster) and explain where each may be appropriate.
- Evaluate reports and media articles where information has been collected, summarised and interpreted (e.g. phone-in polls, popular scientific magazines, news items on sports, politics, economy).
- Discuss such questions as: When is it appropriate to use a sample? When is the sample size large enough? What sorts of data displays are most appropriate for the data in question?
- Design and conduct simple experiments.

D5 undertake exploratory data analysis

Possible activities:

- Summarise and interpret data using appropriate visual representations (e.g. back-to-back stem-and-leaf plots, side-by-side box-and-whisker plots, scatter plots).
- Summarise and interpret data, using appropriate measures of location, spread and correlation.
- Use exploratory data analysis to solve problems (e.g. investigate the distribution of letters in various languages and use to solve codes or to redesign the rules of word games so they can be played in different languages).
- Determine experimentally the percentage of observations lying within $x \pm ks$ where k is a constant and s is standard deviation.
- Plot a scatter diagram from experimental data and interpret the diagram in terms of the real situation.
- Fit lines to data using a variety of methods (e.g. piece of cotton, 2-means method, 3-medians method, least squares method).
- Discuss relationships between variables and whether these imply cause and effect (e.g. height and weight, the number of hotels and churches in country towns, smoking and lung cancer).

STATISTICAL INFERENCE

Experiences in statistical inference should be provided which enable students to:

D6 examine various sampling methods and the relationship between samples and populations

Possible activities:

- Compare various sampling methods with census data collection, considering such matters as cost, convenience, intrusiveness and the reliability of data.
- Investigate the role of random sampling in quality control (e.g. control charts, acceptance sampling).
- Determine the optimal size of a random sample, justifying one's decision and relating sample size and standard error.
- Discuss the misconception that the bigger the population the bigger the sample needs to be.
- Discuss questions of the kind: Given a particular population, what can be said about a sample chosen at random from the population? Given a particular sample, what can be said about the population from which the sample has been (randomly) chosen?

D7 determine and use estimates of population parameters and confidence intervals

Possible activities:

- Make point and interval estimates of a population parameter from sample data.
- Relate confidence intervals to standard errors (e.g. an approximate 95% confidence interval is given by the estimate ± 2 standard errors (se)).
- Use class data to develop an intuitive notion of confidence intervals (e.g. students calculate sample means and standard errors, and examine the proportion of students whose confidence intervals actually contain the population mean).
- Use simulation to draw repeated random samples from a population and study the distribution of sample means. (This could lead to the central limit theorem.)
- Use probability models in practical situations (e.g. finding areas of irregular figures, rock composition, finding breaks in power lines).
- Plan, implement and report the result of 'capture-recapture' investigations, indicating the assumptions made (e.g. estimate the number of fish in a pond).
- Perform an appropriate statistical test of a hypothesis when given a set of data (e.g use a sign test to test a hypothesis on the median of a population).
- Relate statistical concepts to contemporary issues (e.g. in court cases, public opinion surveys).

Algebra

Purpose and scope

To express ourselves effectively about space, number, measurement and chance, we use special language and notations. For example, the use of digits and place value notation enables us to express numbers economically and efficiently, the use of + and x provides efficient symbols for the operations of addition and multiplication. In using a formula to provide instructions, we find that using symbols to replace words makes it simple and, at times, easier to remember, for example, $A = l \times b$ means 'the area of a rectangle of length l and breadth b is found by computing the product $l \times b$'.

As we explore numbers, spatial figures and other relationships, and observe patterns and structures in them, we continually need to develop concepts and language in order to understand and describe them. We also need to develop notations in order to express them and to work with them. Mathematics brings to the study of patterns an efficient and powerful notation for representing generality and variability, and for reducing complexity — algebra. In order to master this notation, the conceptual understandings that underpin it must be developed.

This strand is described under three subheadings: *Expressing generality*; *Functions*; and *Equations*.

Expressing generality

An algebraic representation will often enable us to see generalisations we couldn't see before and can provide a powerful means of showing that a statement will always be true. For example:

Consider the numbers 9, 10 and 11.
Multiply 9 by 11.
Square 10.
Find the difference between the results.
Do the same for 14, 15 and 16.
Try other sets of three consecutive numbers.

As different sets of three consecutive numbers are tested we may become more and more convinced that the result will always be 1. The further question, 'Does this always work?', moves the problem from particular instances to the general, and lies at the heart of algebra.

Algebraic thinking and notation provide a simple way of expressing the generality in the situation, enabling one to show that 'it must be so' and why. In this case, we might choose to represent the three numbers thus: n, $n + 1$, $n + 2$. An algebraic expression can then be formulated and manipulated in order to show that the difference between the product of the first and last terms, and the square of the middle term, *must* be 1, i.e.

$$(n + 1)^2 - n(n + 2) = n^2 + 2n + 1 - (n^2 + 2n) = 1$$

To solve this problem required moving from the special cases provided in the original problem to a more general statement, expressing the problem in a general symbolic form, manipulating the resulting expression, and interpreting the result in terms of the original problem. The development of these skills is a major goal of school algebra.

Algebraic expressions (e.g. 3m + 5) are the basic components or building blocks for algebraic representation. Understanding algebraic expressions and facility with them involves:

- being aware of algebraic expressions as entities in their own right which can be used to represent (or model) some aspect of 'reality';
- recognising that different expressions can be equivalent, that is, represent the same thing;
- recognising that algebraic expressions are built up from smaller components and can be 'unbuilt';
- knowing the basic conventions of algebraic notation (e.g. that 2 x m and m x 2 are written as 2m) which determine how expressions are built up;
- deciding how to manipulate and rearrange expressions and to what purpose.

Students should understand that it is possible to operate on an algebraic expression independently of its meaning and why one might wish to do so.

Functions

In describing patterns and relationships observed in the 'real' world, and in the 'mathematical' world of space, number, measurement and chance, the idea of a functional relationship between two quantities (or variables) is of fundamental importance. An electrician may calculate her charges by adding a fixed amount, say $40, to the number of completed quarter-hours of work multiplied by $10. To a given value of the variable time (i.e. number of quarter-hours) there is associated exactly one value of the variable charge (i.e. cost for service). This is a defining characteristic of a functional relationship. In this case, we say that 'charge' is a *function* of 'time'.

Students should gain wide experience of functional relationships where variables represent different kinds of things (e.g. location, number of cubes, time, number of people, height, income). They should investigate formulae taken from home and work contexts and those developed in all of the other strands and understand them as everyday examples of functional relationships.

Functions can be described by verbal statements, tables of input-output values, graphs or formulae. Each of these kinds of 'descriptions' can contribute to a fuller understanding of the nature of the relationship between the variables. Students should develop an understanding of each and be able to translate from one to another. The full power of mathematical analysis is only available, however, when relationships are represented by a symbolic expression (or formula). In the case above, the function could be represented by the expression $C = 40 + 10n$, where C is the charge in dollars, and n the number of completed quarter-hours. Such mathematical formulations are more precise and concise than everyday language. The expressions can be studied independently of the context from which they came and can be manipulated in order to find: precise quantitative information about the function; other relationships between the variables; and relationships between this function and others.

The qualitative aspects of a function can often be best understood when illustrated by graphical representations. Thus, drawing a graph of a travel function enables one to gain a feel for the relationship between distance and time. The graphics capabilities of calculators and computers enable graphs to be drawn and manipulated quickly and easily. This will decrease the need for paper-and-pencil point plotting but some curve sketching skills will continue to be necessary in order for students to use the technology effectively. Graphics technology should enable students to explore the features of functions more fully than has been possible in the past, leading to an increased understanding of the shapes of families of functions and to an ability to visualise the effects of transformations on those functions (e.g. compare $f(x)$ with $f(x + a)$, $f(-x)$, $f(ax)$).

Equations

Another use of algebraic expressions, and of their rules of operation, derives from the mathematical formulation of situations in which the values of an 'unknown quantity' are required to be found from information about the quantity. An example of such a use is when we represent the statement 'the sum of two numbers is 10 and their product is 24', as:

$$x + y = 10$$
$$xy = 24$$

Originally the unknown quantities were found by methods for which the language and notations were complicated, and this made the actual methods hard to understand or explain. Algebraic notation developed in the last 400 years has helped to improve this.

Algebraic notation assists us to formulate equations and to solve them efficiently. For example, the distance travelled for a particular type of journey may be a function of time, t, and the relationship represented by: $d = t^2 - 2t + 5$. We may wish to know how long it would take to travel a distance of 85 km. Formulated mathematically, for what value(s) of t is $t^2 - 2t + 5 = 85$? In this case two different values for t satisfy the equality, one positive and one negative. Interpreting these values in original context would lead us to discard one of the values as not being meaningful (see **Choosing and using mathematics**). We may also be interested in comparing journeys in two different kinds of vehicle, the travel of the second vehicle being represented, for example, by $d = 6t - 10$. This may lead to the question, 'If the vehicles start together and take the same route, at what times will they be at the same place? Expressed mathematically, for what value(s) of t is $t^2 - 2t + 5 = 6t - 10$?

Students will need to develop various ways of thinking about equations if they are to formulate equations and have access to alternative strategies for solving them. Initially, the emphasis in equation solving should be placed on guess-and-check iterative approaches using calculators and finding approximate solutions to equations from graphs and equations. These are more general than, for example, factoring solutions and expose the essential meaning of 'a solution' more fully. They also emphasise that *applying* mathematics is often about finding 'good enough' answers for the circumstances. If students are to gain access to the full power of mathematics, however, they will need to develop some of the important analytic techniques for solving equations.

ALGEBRA in BANDS A /B

Overview

Basic patterns of algebraic thinking are developed during the primary years. For example, the notions of 'in general', 'variation' and 'function', and 'unknown quantity' are implicit in much of the work of every other strand. These ideas should be fostered by emphasising algebraic thinking throughout bands A and B. The suggested experiences included under algebra have *all* been taken from work in other strands; they are repeated here to highlight the continuity in mathematical development through the years of schooling. Children should not be expected to generate symbolic algebraic descriptions or formal terminology during the primary years.

Children are likely to have their earliest algebraic experiences when, for example, they count a number of counters into a container, take out a handful and then are able to state whether there are more in the container and, if so, how many. Such experiences are important, not only to gain facility with number facts but also because they form the basis of algebraic thinking.

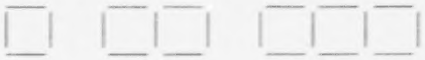
A child may correctly solve [ ] – 7 = 3, and ten others like it, by trial and error, or by recognising the subtraction fact, thereby gaining important number practice. Algebraic thinking occurs when the *attention* shifts from actual manipulation of numbers to the general relationship between the numbers. Using a computer program which provides many questions like [ ] – 7 = 3 can encourage children to find a general strategy for finding the unknown value (e.g. [ ] = 3 + 7). Cases like [ ] – 236 = 304 can be used to 'force' the use of a strategy. Children should be encouraged to explain their rule and test it on other cases, using a calculator to relieve the computational burden. The essentially algebraic feature of this process is in recognising that there is something *general* here (e.g. after a while I saw that I just needed to add the two numbers together and that must give the missing number).

Interpreting operation and equality signs appropriately is very important. Children who interpret the equal sign in 7 + 3 = 10 as 'makes' and in 9 – 5 = 4 as 'leaves' may have difficulty with statements such as [ ] = 4 + 3. They may believe that the 'answer' must come after the equal sign and not find statements like 4 + 3 = 3 + 4 acceptable, suggesting that a 7 is needed. When asked to find the unknown value in [ ] + 5 = 12, they may know that placing a 7 in the box 'makes the sentence true' but believe that 12 is the *answer*.

If these interpretations persist they can inhibit children's mathematical development. For example, older students may feel the need to 'do something' with expressions such as 4 + a or m + n and condense them to 4a or xy, and they may be unable to make a sensible meaning for statements such as 4a + 5 = 2a + 9. In order to understand such expressions, students need to recognise the equals sign (=) as indicating the equivalence of expressions. During bands A and B, children need to experience mathematical statements of equality arising in a variety of contexts. They may record 3 + 8 = 5 + 6 as a statement about weights which balance on a beam or as a statement of equivalence of lengths. Also, they may build up mathematical statements of equality for themselves. For example, they may be asked to complete the statements like this by placing another product on the right side: 2 x 9 = [ ] x [ ].

They may then do the same by replacing one side with a sum or difference. Over time they should generate more complex expressions of equality (e.g. $(12 + 8) \times 3 = (6 \times 8) + 12$), and check each other's efforts. They may even mask one number in an expression and invite their peers to work out what it must be.

During the primary years, children will investigate pattern and variation in areas of mathematics with which they are comfortably familiar, in particular, whole number sets and spatial configuration. Children can work with a variety of numerical and spatial patterns, and find ways of expressing the generality inherent in them. Consider the following match pattern:



Children could identify the pattern as 'You start with four for the first square and then add on three for each extra square' or 'The number of matches needed is three times the number of squares plus one'. Other ways of describing the pattern are also possible, leading children to recognise that different descriptions can fit the same spatial arrangements.

Children vary considerably in the extent to which patterns are obvious to them. Clearly, it is necessary to build on children's particular areas of strength. Some will develop more rapidly if they begin with spatial patterns and then extend their activities to include number patterns; for others the reverse will be true.

The expression of patterns in a general form (e.g. each time you multiplied our number by two and then added one) is one aspect of the development of the function concept. Another way of developing functional concepts is through the investigation of relationships which may be based on familiar daily activities, for example, relating the time of day to level of hunger (very full, comfortably full, not hungry, a little bit hungry, fairly hungry, starving), or in measurement activities. The relationships can be described in words, orally or written, and also graphically, using both informal sketch graphs and those based on measurements made. The aim is for children to understand that graphs are intended to help us get a feeling for the relationship between things that vary.

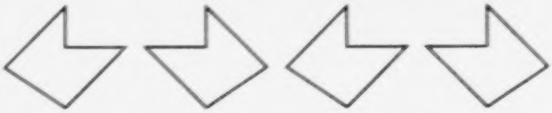
EXPRESSING GENERALITY

Experiences in expressing generality should be provided which enable children to:

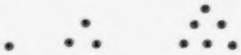
AB1 use verbal expressions (oral or written) to describe and summarise spatial or numerical patterns

Possible activities:

- Describe simple spatial sequences and predict elements (e.g. sequence of shapes).



- Create, explain and compare rules for continuing a spatial sequence, such as:



- Describe simple number sequences verbally and predict elements (e.g. 3, 6, 9, ..., ..., or 251, ..., 273, ..., 295, 306).
- Describe verbally patterns arising in the 1–100 chart and multiplication and addition tables (e.g. investigate multiples of 9 and explain the pattern).
- Investigate verbal expressions which may be ambiguous (e.g. three oranges and apples).

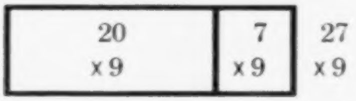
AB2 make and use arithmetic generalisations

Possible activities:

- Make and explain an arithmetic generalisation (e.g. for all the products in the 9 times table, 9, 18, 27, ..., 81, the sum of the digits is nine; I think this happens because ...).
- Investigate answer patterns associated with an operation, e.g.:

$$\begin{array}{cccc} 16 & 26 & 36 & 46 \\ \underline{-9} & \underline{-9} & \underline{-9} & \underline{-9} \end{array}$$

  and predict answers to related questions (e.g. 86 – 9).
- Draw rectangles to demonstrate the distributive property of multiplication over addition for specific cases (e.g. show 27 x 9 = 20 x 9 + 7 x 9 using folded paper or grid paper, and state a generalisation).



FUNCTIONS

Experiences with functions should be provided which enable children to:

AB3 represent (verbally, graphically, in writing and physically) and interpret relationships between quantities

Possible activities:

- Sketch informal graphs to model familiar events such as variations in hunger through the day.
- Given a sketch graph (e.g. of the depth of water in the farm water tank), write a story about it.
- Use a calculator as a simple function machine and graph the input–output pairs.
- Play one- and two-stage 'guess my rule' games and describe the rule verbally (e.g. whatever number I said, you added 1 and then multiplied by 2).
- Represent points on a graph which obey qualitative constraints (e.g. Mara is taller but younger than Sven).
- Represent points on a graph which obey quantitative constraints (e.g. imagine a rectangle with an area of 24 squares — plot a point to show the length and breadth of your imagined rectangle; imagine another ...; are any lengths impossible?).
- Sketch and interpret tables and graphs arising from measurements (e.g. the circumference of circular lids of different diameters, the length of a shadow at different times of the day).

EQUATIONS

Experiences with equations should be provided which enable children to:

AB4 construct and solve simple statements of equality between quantities

Possible activities:

- Use measuring equipment to undertake investigations leading to statements of equality (e.g. show 4 x 3 = 2 x 6 with a variety of material).
- Build up arithmetic statements of equality and check with peers, e.g.:

  $$(4 \times 3) + 10 - 2 = (7 \times 2) + 6$$

- Determine 'hidden' numbers in arithmetic situations (e.g. count 15 counters into a bag, take out a handful, count them and then say how many are left in the bag).
- Given sets of one-stage numerical 'equations', develop a solution strategy (e.g. solve several like [ ] + 13 = 20 and then explain a way that always works; test it on some like [ ] + 382 = 513).
- Play 'think of a number' games to improve mental arithmetic skills, and for simple cases have children investigate and explain verbally why it 'always works'.
- Find missing numbers in calculations (e.g. *** − 387 = 146) and discuss and compare strategies.
- Determine 'hidden' quantities in measurement situations (e.g. I fitted 4 of these rods plus 2 units along the desk, she fitted 3 of the rods plus 6 units — from that we worked out that the rod must be 4 units long).
- Work in groups to solve measurement problems using 'guess, check and improve' (e.g. if a square sheet of cardboard has an area of 1391 cm^2, how long must its side be? use only a four-function calculator — no square root key — and discuss how accurate the answer needs to be).

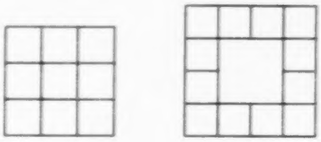
ALGEBRA IN BAND C

Overview

Students need considerable facility with the observation and expression of generality prior to the introduction of algebraic symbolism. Experience within earlier bands will have provided them with some of the necessary experience with numerical and spatial patterns and relationships but more will be needed during band C.

Students must first be able to 'see' a pattern. Having identified it, they should continue to develop ways of describing the pattern mathematically. Many students find the verbal expression of generality difficult. This may be particularly so for some NESB children, even if they have surface facility with English. The expression of generality often requires quite complex sentence structure and logical connectives but, with help, students will develop the necessary language skills.

When students have experienced the need to express generality they should be assisted to find simple and concise ways of doing so. This should begin with progressively shortened ways of recording generalisations. For example, students may build frames around squares:



and say, 'To work out the number of small squares for the frame you need to multiply the number of squares along the side of inner square by two and then add four to the result'. Later they may say, 'If a square has sides of some number, the frame will need 4 times that number plus 4', or provide a shorthand version, '4 x number + 4'. It is at this stage that the use of letters to denote numbers or quantities can be introduced.

It is essential that students understand that letters stand for numbers, not for objects, and that clear distinctions are made between the use of letters in algebra and other uses (e.g. with measurement units, 100c = $1, but 'the number of cents = 100 x the number of dollars' might be written as $c = 100d$).

Many of the standard conventions for writing algebraic expressions, such as:

$y + y + y + y = 4y$

$a \times b = ab = a.\ b$

$w \times w \times w = w^3$

$a \div b = a/b$

$6 \times a = a \times 6 = 6a$

may seem arbitrary to students. For example, why do we write $6(n + 4)$ rather than $(n + 4) \times 6$ or $(n + 4)6$? If students are not to develop the view that mathematics is about the arbitrary manipulation of symbols, they must be helped to distinguish between those aspects of mathematics which are inherently meaningful (such as distributing multiplication over addition) and those which are conventions only.

During band C, students should establish and come to recognise common identities, such as:

$$a^2 - 1 = (a - 1)(a + 1)$$
$$a^2 - b^2 = (a - b)(a + b)$$
$$(a + 1)^2 = a^2 + 2a + 1$$
$$(a + b)^2 = a^2 + 2ab + b^2$$

from spatial and numerical investigations and apply them in various contexts.

Spatial arrangements can help students see that two expressions *must* be equivalent and this can give meaning to algebraic manipulation. They should learn to manipulate simple algebraic expressions which occur in meaningful contexts. For example, students may produce different expressions for the general term in a numerical or spatial sequence or for describing particular relationships, and then rearrange and manipulate the expressions to show their equivalence.

Students should use notational conventions, additive and multiplicative inverses, and the distributive property of multiplication over addition in order to rearrange terms in algebraic expressions, remove brackets and take out common factors. During the compulsory years of schooling, however, they need not develop algorithms for factorising trinomials into binomials. The distributive property or the identities listed earlier can be used directly to find the product of two binomials, but repetitive practice of a computational algorithm for doing so is unnecessary. A major purpose of manipulating algebraic expressions is to express them in more convenient and immediately useful forms. The routine manipulation of algebraic expressions in isolation from any context of their use should not be the major focus of school algebra.

The study of functions should begin with a sampling of relationships familiar to students. Students should use sketch graphs to model these relationships drawn from their own daily experiences (e.g. mood at different times of the day) and from their understanding of other forms of variation (e.g. speed of a car travelling around a race track). Their sketches should reflect the difference between discrete and continuous data (although those terminology are not expected), and between essentially deterministic and probabilistic situations. An example of the latter would be to produce rough scatter diagrams to show how body weight might relate to skill in various sports (e.g. body weight would show a positive relationship with weights lifted; would we expect the same for body weight and height jumped?). Students should informally shade regions to represent various constraints (e.g. if the floor of a house is a rectangle and has an area of between 9 and 16 squares, shade a region on a graph to show all possible room dimensions).

Direct access to a function grapher should enable students to explore families of functions more thoroughly than is possible when all graphs have to be drawn by plotting points by hand. Thus they should gain a wide experience base fairly rapidly, develop an intuitive grasp of the general shapes of particular kinds of functions (e.g. quadratic, cubic, exponential, reciprocal) and be able to visualise the effects of the transformations (in particular, of translation and reflection) on functions. With this intuitive and visual basis students should be able to analyse the effect of parameter changes on graphical and algebraic forms of functions.

During band C, students should formulate equations from a range of numerical, spatial and measurement contexts. They should learn to solve:

- equations of the form $xy = 0$, by concluding that either $x = 0$ or $y = 0$;
- equations of the form $f(x) = 0$, using 'guess, check and improve', successive approximation (iteration), graphical iteration and 'backtracking';
- linear equations, by using properties of equalities (e.g. $A = B$ if, for any $C \neq 0$, $A \pm C = B \pm C$, etc.), that is, 'doing the same thing to both sides';
- simple inequalities using backtracking, graphing and by using properties of inequalities (e.g. $A<B$, if, for any C, $A \pm C < B \pm C$, etc.);
- pairs of simultaneous equations with an emphasis on graphical and iterative approaches, although simple cases such as $2n + m = 7$ and $n - m = 7$ should be solved by elimination.

Finally, students should realise that once they have a mathematical formulation of a problem, in this case an equation, they can deal with that formulation independently of the original context. This means that equations can be solved by a range of strategies which may not relate directly to the original situation.

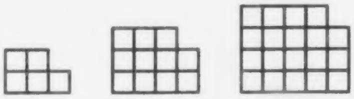
EXPRESSING GENERALITY

Experiences in expressing generality should be provided which enable students to:

C1 express a generalisation verbally (orally and in writing) and with standard algebraic conventions

Possible activities:

- Investigate verbal expressions which may be ambiguous (e.g. three oranges and apples) and restate to remove the ambiguity and relate to the development of precise language in algebra (e.g. distinguish between $3(a + b)$ and $3a + b$).
- Describe verbally the relationship in data generated from structured situations, e.g.:



- Compare different algebraic expressions for the relationship in structured data where the correctness of both expressions can be justified from the numerical or spatial pattern (e.g. the number of squares in the pattern above is $k(k + 1) - 1$ but another way of looking at it gives $k^2 + k - 1$).
- Explain possible ways for continuing the elements in a numerical or spatial sequence (recognising that more than one sequence may be perceived).
- Explain a rule which would generate each element of a spatial sequence knowing the position of the element.
- Play 'guess my rule' games, and build general expressions for the rule, first verbally (e.g. whatever number I said, you added 7, multiplied by 2 and then added 5), later in shortened forms (e.g. [my number + 7] x 2 – 5 = your answer; 2[n + 7] – 5 = your answer).
- Compare different ways of expressing the same rule when playing 'guess my rule' (e.g. I think you added 2 and then multiplied by 3 and then added another 1 but Tran thinks you multiplied by 3 and then added 7 — who is right?).
- Invent 'think of a number' tricks and explain verbally why the tricks work.
- From a verbal expression of a rule of thumb re-express the rule using an algebraic expression and a calculator key sequence (e.g. a rule of thumb for getting an idea of the temperature in degrees Fahrenheit, if you know it in degrees Celsius, is to double the degrees Celsius and add 32).
- Use algebraic expressions to represent numbers in specified sets (e.g. odd numbers can be written as $2n + 1$ where n is a whole number).
- Translate verbal statements into algebraic statements (e.g. the rectangle is twice as long as it is wide) and compare alternative algebraic expressions.
- From an algebraic expression (e.g. (3 x m) + 4) write a 'story' in which the expression makes sense.
- From a set of input and output numbers for a 'function machine', write an algebraic expression for the rule that produced the output numbers.
- Investigate calculator key sequences which are ambiguous because they give different results on the two types of calculators, and rewrite the expressions to remove the ambiguity.

C2 generate elements of a pattern from a verbal or algebraic expression of its rule

Possible activities:

- From a starting number and a rule expressed in everyday language, find the first few elements (e.g. start with 3, multiply any number in the sequence by 2 and then subtract 1 to get the next number).
- From a starting shape and a rule for continuing, find successive elements of a sequence.
- Investigate the effect on the sequence of changing the rule in various ways.
- From a symbolic expression of the general term of a sequence, generate the terms of the sequence.
- From a set of 'input numbers' and a verbal or symbolic rule generate a set of output numbers (e.g. given each month's usage of electricity and the formula for calculating the cost, determine the cost for each month).
- From a rule in the form of an operating instruction for a 'function machine', choose input numbers and generate output numbers.
- From a rule in the form of an operating instruction for a 'function machine', choose output numbers, predict what the input numbers must have been and validate using a calculator as the function machine.

C3 develop and apply algebraic identities involving the use of the distributive property of multiplication over addition and index laws for integral powers

Possible activities:

- Investigate in order to make explicit the arithmetic rules which underlie standard computational algorithms, e.g. 23 x 35; ${}^{3}/_{8} + {}^{5}/_{8} = {}^{3+5}/_{8}$
- Investigate spatial arrangements and diagrams leading to generalisations such as the following:

  Perimeter = 2(L + B) and Perimeter = 2L + 2B
  $2b + 8 = 2(b + 4)$
  $a^2 - 1 = (a-1)(a+1)$
  $(a+1)(a+1) = a^2 + 2a + 1$
- Demonstrate the correctness of algebraic identities in numerical situations by substitution.
- Develop identities through investigations of number patterns. For example, find a simpler way of finding the elements in this table:

a	0	1	2	3	4
$a^2 + 2a + 1$					

- Use a calculator to explore repeated multiplication.
- Investigate scientific notation as used on a scientific calculator.
- Use a calculator to investigate sequences such as:

  $\ldots 2^{-4}, 2^{-3}, 2^{-2}, 2^{-1}, 2^{0}, 2^{1}, 2^{2}, 2^{3}, \ldots$
  $\ldots 3^{-4}, 3^{-3}, 3^{-2}, 3^{-1}, 3^{0}, 3^{1}, 3^{2}, 3^{3}, \ldots$
  $\ldots 4^{-4}, 4^{-3}, 4^{-2}, 4^{-1}, 4^{0}, 4^{1}, 4^{2}, 4^{3}, \ldots$

  and make generalisations about indices (e.g. the relationship between 2^{-2} and 2^{2}).

C4 manipulate algebraic expressions for a purpose making use of notational conventions in algebra, the distributive property of multiplication over addition, and inverses of addition and multiplication

Possible activities:

- Given arithmetic statements (e.g. $7 + 3 = 10$), write equivalent expressions using the same numbers (e.g. $7 = 10 - 3$ and $10 - 7 = 3$).
- Invent 'think of a number' tricks, write algebraic expressions for the rule, and manipulate the expression to show why the trick always works.
- Given a pair of algebraic expressions, informally discuss whether they are definitely equal, possibly equal or not equal (e.g. $x^2 + 4x$ and $x(x + 4)$; $(a^2 + b^2)$ and $(a + b)^2$).
- Use the distributive property and rearrangement of terms to show the equivalence algebraically of alternative expressions which students have developed to describe a numerical or spatial pattern.
- Investigate the truth or falsity of general statements about numbers and justify conclusions using a general algebraic argument (e.g. 'an odd number multiplied by an odd number always gives an odd number').

FUNCTIONS

Experiences in functions should be provided which enable students to:

C5 identify variation in situations and use the idea of variable

Possible activities:

- Carry out structured spatial or measurement activities where variation is evident and identify the variables in the situation (e.g. build rectangles with unit squares and investigate dimensions, areas and perimeters).
- Identify the variables in published data (e.g. use a list from a hardware store of timber dimensions available and prices).
- Give oral descriptions of familiar situations in which variation is inherent and identify the variables (e.g. when I jog, I usually start off slowly and build up a comfortable speed and then slow down gradually as I get near the end of my run).
- Interpret verbal explanations of situations where variation occurs, identify the variables and explain in own words (e.g. study an extract from a social studies text explaining the relationship between standard time and latitude).
- Given extracts from standard textbook exercises (e.g. next year Paul will be three times as old as Ann will be then), write expressions involving a variable and compare alternative approaches (e.g. compare the statements obtained if one lets the variable t be Ann's age now in years, or the variable m be Paul's age next year in years).

C6 draw freehand sketches of and interpret graphs which model real phenomena qualitatively

Possible activities:

- Draw and interpret bivariate point graphs which represent situations qualitatively (e.g. draw a graph which shows that Rina is older than Rachel, but Rachel is taller than Rina).
- Compare qualitative graphs of the same familiar situation and choose the one which best represents the situation (e.g. these graphs were sketched to show the relationship between how many people there will be at the school canteen and the time of day; discuss what each means and say which is most realistic and why).
- Sketch graphs to model situations qualitatively (e.g. temperature throughout the week) and discuss and redraft the graph until it adequately reflects the situation as perceived.
- Use sketch graphs to model situations on the basis of a verbal description (e.g. about my learning curve when I learnt to play the guitar) or a picture (e.g. given a roller coaster draw a speed/time graph).
- Use sketch graphs to model situations on the basis of a verbal description in which comparative language (e.g. more) is used.
- Use sketch graphs of regions to model situations of 'one or many' (e.g. I have at most 20 days holiday a year and I can take the days in one or two lots; show all the possible combinations of days).
- Sketch graphs to model phenomena (e.g. water dripping into bottles, cars on a race track) based on observation or qualitative analysis.
- Interpret sketch graphs which represent unfamiliar relationships (e.g. given a graph showing the spread of a disease over a period of time, explain the situation verbally).

C7 use graphs to model real situations and make predictions including those based on interpolation, extrapolation, slope and turning points

Possible activities:

- Find data from realistic situations and from structured numerical and spatial situations, organise (tabulate) information into ordered pairs and plot the points.
- Produce and interpret a scatter graph of real data (e.g. heights of mothers and daughters, age and weight of children).
- Produce graphs of physical situations based on direct and indirect measurement and on secondary data sources.
- Consider statements of inequality involving only one variable (e.g. first $12 < 16 - [\ ]$ and later $12 < 16 - b$) and list replacement values, plot observed replacement values on a number line, and generalise to graph all replacement values.
- Graph curves and regions in the plane to represent structured situations involving constraints. (e.g. imagine a rectangle of 36 squares; plot a point to show the rectangle imagined; another ...; plot some rectangles which are too big in a different colour).
- Graph regions in the plane to represent realistic situations involving comparisons (e.g. if the combined income of a couple is $60 000 or more for a particular year, they are not eligible for the government child allowance the following year).
- Interpret and draw situations which can be represented by travel graphs and investigate gradient–speed relationship.
- Use appropriate computer software to investigate graphs of situations in which constraints can be varied, asking 'what if ...' questions.
- Use interpolation, extrapolation, slope and turning points to interpret and make predictions from graphs of social or physical phenomena.

C8 recognise algebraic expressions of linear, reciprocal, quadratic and exponential functions and the graphs which represent them

Possible activities:

- Investigate practical situations which are approximated by linear, quadratic, exponential (e.g. growth and decay) and reciprocal (e.g. distance–time graphs) functions.
- Use scientific calculators to generate a variety of functions, plot the points and compare the shapes of the graphs with graphs previously produced for real situations.
- Investigate families of lines, produce the graph of a line given its slope and intercept, read intercepts and determine slopes from graphs.
- Using function graphers, investigate families of functions, that is, consider the relationship between $f(x)$ and $f(x + a)$, $f(-x)$ and $f(ax)$ and interpret in terms of real phenomena.

C9 use algebraic expressions (formulae) to model situations and make predictions based on the general characteristics of the formulae

Possible activities:

- Use physical materials to create patterns which can be described by simple linear function rules.
- Given a set of 'input numbers' and a rule, generate a set of output numbers (e.g. given ages of several children and the formulae for calculating the dose of medication given

the age, calculate the dosage for each child).

- Generate data from structured situations and use difference patterns and ratios to derive a linear, quadratic, reciprocal or exponential formula relating the variables.
- Graph real data relating two measured variables (e.g. cooling of a cup of coffee over time), fit a curve 'of good fit' by observation and derive a formula relating the variables.
- Make measurements of related variables in actual situations, and represent the data in the form of graphs or formulae in order to understand the relationships between the variables.
- Investigate the various relationships in a set of data relating to a real situation (e.g. various window dimensions and cost of replacement), informally fit curves to the data and develop formulae relating the variables.
- Consider the appropriateness of formulae in particular real situations.
- Given a verbal explanation of a relationship (e.g. an extract from a social studies text explaining the relationship between standard time and latitude) express the relationship using an algebraic expression, a calculator key sequence and a graph or diagram.

EQUATIONS

Experiences in equations should be provided which enable students to:

C10 formulate equations and inequalities in a range of situations

Possible activities:

- Construct statements of equivalence of two arithmetic expressions and check with peers, e.g.:

$$(2 \times 7) - 5 + 2(4 + 9) = 5 + (5 \times 4) + 10$$

- Construct statements of equivalence of two arithmetic expressions, mask one number and ask a peer to find the unknown value.
- Play 'guess my rule' in reverse, and express the reverse task verbally (e.g. the rule was 'multiply the input number by 3 and add one', and the output number was 16, so I have to find what number multiplied by 3 and added to 1 gives 16) and then express it algebraically (e.g. the rule was $3n + 1$, the output number 16, and so I have to find n when $3n + 1 = 16$).
- Experiment with masked weights and a balance beam and set up equations involving the masked weight.
- Given a meaningful functional relationship (e.g. the formula relating temperature in Fahrenheit and Celsius is $F = {}^9/_5\,C + 32$), write equations (e.g. today it was 27°C; to find what that is in Fahrenheit, we need to solve $F = {}^9/_5 \times 27 + 32$).
- Investigate 'real' situations in which two algebraic expressions are generated, compare the expressions leading to informal investigation of equations and inequations (e.g. different newspapers offer different rates to sellers — express payments as $8N + 50$ or $10N$ — leading to a discussion of conditions under which one or the other is preferable, i.e. when is $8N + 50 = 10N$? when is $8N + 50 > 10N$?).

C11 solve equations choosing an appropriate technique from 'guess, check and improve', successive approximation, graphical iteration, 'backtracking' and 'do the same thing to both sides', and interpret solutions in the original context

Possible activities:

- Play 'guess my rule' in reverse (e.g. two people are given a rule, one chooses a number, performs the rule on it and tells the other the result; the second person then determines the chosen number).
- Play 'guess my rule' in reverse: explain *how* the original number can be found from the rule and demonstrate this by 'stripping' or 'backtracking' algebraically.
- Use calculator key-stroke notation as a device to help 'strip' or 'backtrack' an algebraic equation.
- Use a calculator to solve a range of equations using 'guess, check and improve' and discuss strategies for doing this efficiently leading to the idea of iteration.
- Use iterative techniques to solve problems arising in other strands (e.g. find a calculator approximation to $\sqrt{5}$, following a discussion of Pythagoras' theorem).
- Use iterative techniques with a calculator to solve equations, considering techniques for ensuring that all solutions are found (e.g. express: $x^2 + 4x - 10 = 0$ as $x(x + 4) = 10$).

- Use a function grapher to find approximate solutions to an equation, zooming to improve accuracy.

C12 solve simple inequalities, choosing an appropriate technique from 'backtracking', graphing and properties of inequalities

Possible activities:

- Informally investigate values for which it is true that one expression is greater than (or less than) another (e.g. investigate whether 3x is always less than 4x).
- In a realistic situation involving inequality (e.g. if the combined income of a couple is $60 000 or more for a particular year, they are not eligible for the government child allowance the following year) decide the status of particular pairs of values with respect to the inequality (e.g. for particular pairs of salaries decide whether the couple would or would not receive the child allowance the following year).
- Shade regions of the plane to show the effect of a constraint which uses comparative language (e.g. the room has to be a minimum of 9 m^2 but it shouldn't be any bigger than 16 m^2).
- Shade regions of the plane to show the effect of sets of constraint which use comparative language (e.g. subject to the above constraints and also: the width must not exceed 4 m, and the length must be less than twice the width) and interpret the graph.

ALGEBRA in BAND D

Overview

During band D, the emphasis in algebra should continue to be on the use of algebraic notions to express generality and the development of algebraic methods for solving problems. Students should continue to investigate real or structured situations, observing patterns and relationships amongst variables, and represent them algebraically and graphically.

Students should extend their repertoire of forms of representation. They should use a range of types of algebraic expressions including, for example, rational and fractional expressions and indices with rational and negative powers. A wide range of situations can be expressed in matrix forms and students should experience at least some of these. They should formulate relationships either by expressing a general term or iteratively, making decisions about the most appropriate form in the context of use.

Students should recognise some of the range of situations in which we manipulate and rearrange algebraic expressions and do so for a specific purpose. For example, the direct computation of:

$$3n^2 + 2n - 3 + n^2 + 5n - 4 - 6n$$

for $n = 20$, is helped considerably by simplifying the expression to:

$$4n^2 + n - 7 \text{ or } n(4n + 1) - 7$$

even if the 'number crunching' is to be performed with a calculator. Proving identities, operating on functions and solving equations will require control of the notational system (that is, manipulative skill) and students should have the skills needed to support their learning in other strands and their problem solving generally. For some students, this will include manipulation of rational and irrational indices and surds. Some will also study operations on matrices but should use calculators or computers to perform routine computations and manipulations.

During band C, students will have informally investigated a wide range of discrete and continuous functions and made a more detailed study of families of linear, quadratic, inverse and exponential functions. During band D, they should extend this range considerably. They may study, for example, logarithmic functions, general polynomial functions, absolute value functions, piecewise and step functions, and the conic sections. The particular functions chosen may vary but the emphasis should be on the use of these functions to model and hence to understand phenomena. *All* students should investigate a variety of recursive processes, undertaking a sufficiently detailed analysis of certain periodic functions to enable students to model with them.

Students should use computer graphing packages to study the behaviour of functions with respect to asymptotes and limiting values and analyse the effects of transformations of the functions. Some may investigate surfaces generated by functions in two variables.

Students should formulate equations and inequalities, including sets of simultaneous equations and inequalities, from 'real' or simulated situations, and develop a repertoire of analytic, graphical and computational strategies for solving them. Graphing calculators or computer packages may be used to determine approximations for solutions of equations or inequalities to various degrees of accuracy. Some students may also use matrix methods for solving systems of linear equations.

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EXPRESSING GENERALITY

Experiences with expressing generality should be provided which enable students to:

D1 express generalisations, functions and equations algebraically using one or more variables

Possible activities:

- Use Pythagoras' theorem to solve problems.
- Develop formulae in coordinate geometry (e.g. mid-point formulae for line segments).
- Represent situations using pairs of simultaneous equations.
- Given a locus (e.g. a verbal description of a circle or ellipse), develop an algebraic representation.
- Given a verbal description of constraints (e.g. for a design task), express them as a set of linear equations or inequalities.
- Establish meaning for rational indices through geometric or graphical means (e.g. find an expression for the length of the side of a square with diagonal 10 units, or area 10 units ..., graph the integral values of, for example, 8^x and discuss sensible meanings to apply to intervening points, i.e. non-integral powers).
- Construct and interpret simple route matrices to describe given networks and vice versa (e.g. this matrix represent the traffic flow in a city street system with A, B, C and D as corners; which are the one way streets?).
- Interpret the use of matrices in practical decision-making situations (e.g. stock control).
- Represent transformations in two or three dimensions by a matrix and explain the relationship between a transformation and the form of the corresponding matrix.
- Explain the links between vector and matrix representations.
- Represent data in a spreadsheet in matrix form for use in solving equations.
- Write a simple iterative formula to find each term of a sequence given the previous term or terms ($T_{n+1} = T_n + T_{n-1}$).

D2 manipulate algebraic expressions to generate more convenient forms

Possible activities:

- Translate expressions for a function from one form to another to facilitate curve sketching (e.g. express $x^2 - 2x + 7$ as $(x - 1)^2 + 6$).
- Factorise polynomial expressions (e.g. by inspection or the factor theorem) in order to solve equations.
- Manipulate algebraic expressions in order to simplify computation and data entry.
- Change one form of an expression to another for a specific purpose (e.g. from polar to rectangular coordinate representations of a circle).
- Generalise the rules used for manipulating numbers expressed in integral powers to include real powers and express the rules algebraically.
- Carry out matrix computations for a purpose (e.g. to compute total cost from matrices which summarise information about numbers of items and their costs, respectively; to represent compositions of transformations).

FUNCTIONS

Experiences with functions should be provided which enable students to:

D3 identify and express recursion and periodicity in various contexts

Possible activities:

- Explain everyday events as iterative processes (e.g. inflationary spiral, population growth).
- Consider the effect of moving according to a given set of rules (e.g. at each stage turn right and walk one step more than you did before).
- Make physical representations of periodic functions (e.g. students stand, with calculators, on the x-axis of a grid and move on the grid according to instructions such as sin x or sin 2x).
- Design, test and adapt an iterative Logo routine to produce a desired effect.
- Use spreadsheets to investigate the cumulative effect of compound interest.
- Use an iterative formula to find an integral (e.g. using trigonometric functions).
- Iterate a rule for deforming the sides of a figure to produce space-filling curves.
- Use a computer to iterate a complex polynomial function to produce a fractal pattern.
- Identify periodicity in natural events (e.g. tides, wave motion, menstrual cycles) and draw sketches to model the situations.
- Investigate non-circular periodic functions (e.g. cycloids, heartbeats, wrapping functions).
- Express circular movement in terms of trigonometric functions.
- Contrast the forms of different trigonometric functions (e.g. compare sine and tangent, tangent and cotangent).
- Graph a curve with trigonometric parameters (e.g. $x = \sin A$, $y = \tan A$).

D4 recognise and determine important features of families of functions

Possible activities:

- Investigate practical situations which are represented by non-continuous functions (e.g. step and absolute value functions).
- Investigate practical situations which are modelled by logarithmic functions (e.g. sound levels, earthquakes magnitudes).
- Investigate families of functions by translating, reflecting and dilating simple functions (e.g. compare x^2, $x^2 + 1$, $3x^2$, $-x^2$, $(x + 1)^2$, $(-x)^2$, trigonometric functions).
- Classify various formulae involving two variables according to whether they are linear, reciprocal, quadratic, exponential or logarithmic, and predict the nature of the graphs which represent them.
- Interpret a graph in terms of the physical or social situation which it models (e.g. interpret the turning point in a function which arises in a search for minimum area of a rectangle).
- Classify functions on the basis of given attributes (e.g. maxima, minima, points of inflection, axes of symmetry, undefined points).
- Construct families of functions which fit given constraints (e.g. a cubic function with two known turning points).
- Investigate families of logarithmic and exponential functions and identify inverse relationships.

- Identify key features of polynomial functions including, for example, facts about general algebraic form, factorisation, division, factor and remainder theorem.
- Identify features of functions that are important in the identification of composite and inverse functions (e.g. sums, products, quotients, scalar multiples, partial fractions, 1–1 mappings, discontinuity).

D5 recognise different situations which can be modelled by the same function and fit curves to data sets

Possible activities:

- Using graphs from daily newspapers, etc., discuss whether, or how well, classical models fit the relationships.
- Decide whether a linear function approximates a set of data and, if so, fit a line to the data (e.g. by 'eye' or the least squares method).
- Plot the logs of some apparent growth functions (e.g. car registrations, overtime) to produce a near-linear function.
- Fit a trigonometric function to a graph of tide times.
- Use computer packages for curve fitting.

EQUATIONS

Experiences with equations should be provided which enable students to:

D6 formulate equations and systems of equations and solve to appropriate levels of accuracy, making use of graphical, computational and analytical methods

Possible activities:

- Establish analytic procedures for solving quadratic equations (e.g. factorisation, completing the square, or use of a formula).
- Predict the number of roots of a function by inspecting the appropriate equation.
- Use simple iterative processes to find approximate values (e.g. square roots, cube roots).
- Use a calculator to solve equations by 'halving the interval'.
- Examine some simple iterative processes of the form $x_{n+1} = f(x_n)$ as techniques for finding numerical solutions.
- Use calculator or computer software to apply Newton's method to solve equations (e.g. find the zeroes of a polynomial).
- Discuss difficulties inherent in particular functions when using computing technology to solve equations (e.g. if the entries in a matrix are close to those in a non-invertible matrix, 'round off' error may result in the computer not being able to find its inverse).
- Construct tables of values and use graphs to solve equations or systems of equations to appropriate levels of accuracy.
- Write sets of linear equations and solve using matrices.
- Estimate the roots of equations using a graphics calculator, paying particular attention to scales on the axes and the effect of the zooming.

D7 determine feasible regions under sets of constraints

Possible activities:

- Discuss problems to eliminate impossibilities and identify sets of possible solutions where restrictions apply (e.g. identify possible candidates for election to the school council based on rules for membership).
- Sketch regions according to given constraints (e.g. sketch the dimensions of a room given a minimum and maximum area, and a preferred range of ratios between the side lengths).
- Formulate and solve simple two variable problems using linear programming.

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Appendix I

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Commerce and Industry, Northern Territory
Chamber of Manufacturers, New South Wales

Appendix II

Summary of scope statements

This summary was prepared by Dr S Willis on behalf of the project team.

Attitudes and appreciations

Bands A to D

1 a positive response to the use of mathematics as a tool in practical situations
2 the confidence to apply mathematics and to gain knowledge about mathematics they need
3 a willingness and ability to work cooperatively with others and to value the contributions of others
4 a willingness to persist when solving problems and to try different methods of attack
5 an awareness that creativity and initiative are encouraged and valued within mathematics education
6 an interest in and enjoyment of the pursuit of mathematical knowledge
7 that mathematics involves observing, generalising and representing patterns
8 that the economy and power of mathematical notation and terminology help in developing mathematical ideas
9 that conventions, rules about initial assumptions, precision and accuracy enable information to be shared unambiguously
10 that a mathematical model is a simplified image of some aspect of the social or physical environment
11 that rigorous justification of intuitive insights is important
12 that mathematics has its origins in many cultures and is developed by people in response to human needs, both utilitarian and aesthetic

Mathematical inquiry

BAND A

A1 read and write numerical statements, and use and interpret pictorial representations
A2 clarify, use and interpret mathematical terms and phrases
A3 identify and describe regularities and differences
A4 recognise, produce and use spatial and numerical patterns
A5 explain results, conjectures and guesses in everyday language
A6 use problem-solving strategies and compare strategies for solving the same problem
A7 clarify mathematical questions and pose related questions
A8 undertake linked sequences of problems, individually and collaboratively

Band B

B1 draw diagrams and read and write mathematics using simple formulae, pictures, tables and statistical graphs
B2 clarify, use and interpret mathematical expressions and phrases
B3 observe, represent and extend spatial and number patterns
B4 test conjectures in space and number and search for counter-examples
B5 explain conjectures and results, using appropriate mathematical language
B6 use a range of problem-solving strategies, and compare and suggest alternative strategies for the same problem

B7 refine and clarify mathematical questions and pose related questions

B8 undertake structured investigations, individually and collaboratively

B9 use personal and group organisational skills to help in tackling mathematical problems

Bands C/D

CD1 draw diagrams and read and write mathematics using symbols, tables, pictures and graphs

CD2 use and interpret mathematical terms, expressions and phrases orally and in writing

CD3 observe and represent spatial and numerical patterns and identify the same pattern in different contexts

CD4 convince themselves about the validity of conjectures and revise, refine or extend conjectures

CD5 evaluate the validity of arguments designed to convince others of the truth of propositions

CD6 construct arguments designed to convince others of the truth of mathematical propositions

CD7 recognise the effect of assumptions on the conclusions that can be reached

CD8 use a range of strategies to solve problems, compare different strategies and suggest alternative strategies for the same problem

CD9 pose, refine and clarify problems within mathematics

CD10 undertake open-ended mathematical investigations, individually and collaboratively

CD11 develop personal and group organisational skills for tackling mathematical situations

Choosing and using mathematics

Band A

A1 use mathematics in dealing with practical or imagined situations from their experience

A2 represent problems using objects, pictures, symbolic statements or mental images

A3 interpret solutions in the problem context and comment on results

Band B

B1 choose and use mathematical skills to make decisions

B2 clarify and pose problems arising in practical or imagined contexts

B3 represent practical problems using objects, diagrams, symbols or mental images

B4 verify and interpret solutions with respect to the original problem

Bands C/D

CD1 choose and use standard techniques and formulae

CD2 choose and use data collection and analysis tools (including technology) with due regard to the demands of the situation

CD3 choose and use mathematical skills to assist in interpreting information from a variety of sources

CD4 recognise, use and evaluate standard mathematical models

CD5 pose or clarify and refine problems

CD6 formulate simple mathematical models and test by observation or experiment

CD7 adapt mathematical models to provide better fit or make different predictions for testing

CD8 interpret solutions in terms of the original context of the problem

CD9 develop a personal set of managerial procedures to be used in tackling modelling problems

Space

Band A

A1 build structures and make and investigate geometric models

A2 compare and classify objects and describe in everyday spatial language

A3 make and draw a variety of plane figures
A4 compare and classify figures and describe in everyday spatial language
A5 recognise simple translations, rotations and reflections and related symmetries
A6 rearrange, fit together and tessellate figures, and stack and pack objects
A7 make informal use of scale in producing models and drawings
A8 recognise realistic two-dimensional representations of objects
A9 give and follow directions for moving and locating objects
A10 plan, describe and use arrangements in practical settings

Band B

B1 build structures and make geometric models, analysing their cross-sections and nets
B2 compare and classify objects, analyse their shapes and describe in conventional geometric language
B3 make and draw a variety of plane figures using geometric tools
B4 compare and classify figures, analyse their shapes and describe in conventional geometric language
B5 carry out rotations, translations and reflections and recognise and produce related symmetries
B6 recognise congruent figures by superimposition, and informally investigate congruent figures
B7 rearrange, fit together and tessellate figures and stack and pack objects
B8 reduce and enlarge figures and objects and investigate distortions resulting from transformations
B9 produce, interpret and compare scale drawings and maps, and scale models of familiar structures
B10 interpret and produce two-dimensional representations of objects
B11 give and follow directions for moving and locating objects, using angles, compass points and grids
B12 plan and execute arrangements according to specifications
B13 describe, follow and record paths and routes using sketches and simple networks

Band C

C1 construct objects satisfying design specifications
C2 recognise and apply features, sections and nets of polyhedra, spheres, cylinders and cones
C3 draw figures satisfying given conditions, using a variety of geometric techniques and tools
C4 use properties of angles, intersecting and parallel lines
C5 formulate relationships between various polygons and use properties to solve problems
C6 formulate properties of circles and use to solve problems
C7 use translations, rotations and reflections and the relationships between them
C8 identify congruent figures and formulate and apply congruence conditions
C9 enlarge and reduce figures and objects and formulate and apply similarity conditions
C10 interpret and produce two-dimensional representations of objects
C11 give and follow directions for moving and locating objects, using angles, compass points and grids
C12 describe, follow and record regions, paths and routes using networks and loci

Band D

D1 visualise and represent in two- and three-dimensional space
D2 use the language and techniques of coordinate geometry
D3 solve problems, choosing analytical tools from amongst Euclidean, transformation, vector and coordinate geometries, networks and trigonometry
D4 recognise that geometries are invented systems

Number

Band A

A1 compare and match objects and collections
A2 use whole numbers to count and order
A3 recognise, produce and use patterns with whole numbers
A4 use place value concepts to read, write and compare whole numbers
A5 recognise unit fractions (expressed in words and notation) as they occur in practical contexts
A6 use two-digit decimals to express money and measurements
A7 make an appropriate choice of addition, subtraction, multiplication and division of whole numbers for a given context
A8 remember addition facts, and use mental arithmetic to extend these facts
A9 use a basic four-function calculator efficiently
A10 estimate the size of a collection and approximate with whole numbers, measures and money
A11 use efficient paper-and-pencil techniques to add and subtract whole numbers, measures and money, and use personal adaptations to multiply by single-digit numbers
A12 choose computational methods (mental, paper-and-pencil, calculator) and check the reasonableness of results

Band B

B1 use whole numbers and decimal fractions to count and order collections and measures
B2 recognise, produce and use patterns in number
B3 use place value to read, write and compare decimal fractions
B4 interpret common fractions in everyday use and compare fractions for practical purposes
B5 interpret ratios and percentages in everyday use
B6 use numbers in a variety of equivalent forms and translate between forms
B7 make an appropriate choice of addition, subtraction, multiplication or division of decimal numbers, money and measures
B8 remember basic addition and multiplication facts and perform mental computations on whole numbers and money
B9 use a four-function calculator with memory facility efficiently
B10 estimate and approximate computations on decimals, money and measures
B11 use efficient paper-and-pencil methods to add and subtract decimals, money and measures and to multiply and divide decimals by single-digit numbers
B12 choose computational methods (mental, paper-and-pencil, calculator) and check reasonableness of results

Band C

C1 describe the development of counting and numeration systems in various cultures and relate these to social change
C2 count and order collections and measures, using 'standard form' for small and large numbers
C3 distinguish between rational and irrational numbers and understand the significance of recurring and non-recurring decimals
C4 investigate and use number patterns and relationships
C5 use a variety of equivalent forms of numbers (e.g. decimal fraction, common fraction, standard form, percentage) and convert between them
C6 make an appropriate choice of addition, subtraction, multiplication or division of rational numbers for a given context
C7 apply ratio, proportion and percentage in everyday contexts
C8 remember basic addition and multiplication facts and perform mental computation on decimals, including money and measures
C9 use a scientific calculator efficiently

C10 estimate and approximate computations on any numbers

C11 use efficient paper-and-pencil methods to add, subtract and multiply whole numbers and decimal and common fractions, and to divide by single-digit numbers

C12 choose computational methods (mental, paper-and-pencil, calculator, computer) and check reasonableness of results

C13 interpret the language of and deal with matters of personal finance

Band D

D1 identify, derive and compare properties of sets of numbers

D2 represent numbers in a range of forms, and use in solving problems

D3 develop and use models based on sequences and series

D4 estimate numbers and make approximate computations

D5 choose and use appropriate formulae, counting and computational strategies and technologies

D6 understand and apply aspects of personal finance and business-related arithmetic

Measurement

Band A

A1 recognise the attributes of length, capacity, mass, area, volume, time, temperature and angle

A2 make direct comparisons of, and order, objects and events using length, capacity, mass, area, volume, time, temperature and angle

A3 recognise that units are required for measuring and choose units appropriate to the task at hand

A4 use non-standard units for the measurement of length, area, capacity, volume, mass, time and angle

A5 use everyday standard units and measuring equipment to measure length, capacity, mass and time

A6 make estimates of length, capacity, mass, area and time

Band B

B1 measure, compare and order objects and events using length, capacity, mass, area, volume, time and angle

B2 choose appropriate units for tasks by considering purpose, and the need for precision and/or communication

B3 recognise standard metric units of length, capacity and mass and use common metric prefixes to relate units to each other

B4 use standard techniques and tools to measure length, capacity and volume, mass, area, time and angle

B5 place objects in order by estimating their relative sizes and make estimates of length, capacity and volume, mass, area, time and angle

B6 investigate relationships between measures for different attributes and apply to solve problems

Band C

C1 understand the use of metric prefixes to unify measurements in a coherent way and use and design a variety of scales

C2 choose and use appropriate techniques and tools to measure length, capacity and volume, mass, area, time and angle

C3 estimate length, capacity and volume, mass, area, time and angle and make statements about the degree of confidence associated with the estimate

C4 investigate relationships between length, angle, area and volume

C5 make estimates of quantities which cannot be measured directly or conveniently

C6 use measurement formulae to find perimeters, areas and volumes of common two-dimensional and three-dimensional shapes

C7 understand and use the derived measures of density, speed and other rates

C8 use transposition to change the subject of a formula for use in practical settings

C9 interpret and apply information involving time and elapsed time

C10 use similarity and Pythagoras' theorem for indirect measurement in two- and three-dimensions

Band D

D1 choose appropriate techniques and tools to measure quantities to chosen or specified levels of accuracy

D2 use rates to deal with practical measurement tasks

D3 apply similarity relationships involving length, area and volume

D4 use trigonometric ratios to solve problems in two and three dimensions

D5 investigate and use instantaneous rate of change of a variable when interpreting models of situations

D6 use suitable methods of approximation to find areas and volumes of irregular shapes

D7 investigate the use of infinite and limiting processes in integration

Chance and data

Band A

A1 use, with clarity, everyday language associated with chance events

A2 describe possible outcomes for familiar random events and one-stage experiments

A3 place outcomes for familiar events and one-stage experiments in order from those least likely to happen to those most likely to happen

A4 frame questions about themselves, families and friends, and collect, sort and organise in formation in ways in order to answer these questions

A5 represent and interpret information to answer questions about themselves, friends and families

Band B

B1 make statements about how likely are everyday experiences which involve some elements of chance and understand the terms 'chance' and 'probability' in common usage

B2 for random events, systematically list possible outcomes, deduce the order of probability of outcomes and test predictions experimentally

B3 make and interpret empirically based predictions about simple situations

B4 systematically collect, organise and record data to answer questions posed by themselves and others

B5 represent, interpret and report on data in order to answer questions posed by themselves and others

B6 understand what samples are, select appropriate samples from specified groups and draw informal inferences from data collected

Band C

C1 understand and explain social uses of chance processes

C2 construct sample spaces to analyse and explain possible outcomes of simple experiments and calculate probabilities by analysis of equally likely events

C3 estimate probabilities using the long-run relative frequency (that is, empirical probabilities)

C4 model situations and devise and carry out simulations

C5 access, evaluate and interpret information presented in different forms from a variety of sources

C6 systematically collect, organise and record data for practical purposes

C7 summarise and interpret data using visual representations and measures of location and spread

C8 understand the impact of statistics on daily life

C9 understand what samples are and recognise the importance of random samples and sample size

C10 draw inferences and construct and evaluate arguments based on sample data

Band D

D1 use experimental and theoretical approaches to investigate situations involving chance and to determine the likelihood of particular outcomes

D2 generate, use, interpret and investigate properties of probability models

D3 use simulations to model uncertain events

D4 plan, manage and appraise data collection and presentation

D5 undertake exploratory data analysis

D6 examine various sampling methods and the relationship between samples and populations

D7 determine and use estimates of population parameters and confidence intervals

Algebra

Bands A/B

AB1 use verbal expressions (oral or written) to describe and summarise spatial or numerical patterns

AB2 make and use arithmetic generalisations

AB3 represent (verbally, graphically, in writing and physically) and interpret relationships between quantities

AB4 construct and solve simple statements of equality between quantities

Band C

C1 express a generalisation verbally (orally and in writing) and with standard algebraic conventions

C2 generate elements of a pattern from a verbal or algebraic expression of its rule

C3 develop and apply algebraic identities involving the use of the distributive property of multiplication over addition and index laws for integral powers

C4 manipulate algebraic expressions for a purpose making use of notational conventions in algebra, the distributive property of multiplication over addition, and inverses of addition and multiplication

C5 identify variation in situations and use the idea of variable

C6 draw freehand sketches of and interpret graphs which model real phenomena qualitatively

C7 use graphs to model real situations and make predictions including those based on interpolation, extrapolation, slope and turning points

C8 recognise algebraic expressions of linear, reciprocal, quadratic and exponential functions and the graphs which represent them

C9 use algebraic expressions (formulae) to model situations and make predictions based on the general characteristics of the formulae

C10 formulate equations and inequalities in a range of situations

C11 solve equations choosing an appropriate technique from 'guess, check and improve', successive approximation, graphical iteration, 'backtracking' and 'do the same thing to both sides', and interpret solutions in the original context

C12 solve simple inequalities, choosing an appropriate technique from 'backtracking', graphing and properties of inequalities

Band D

D1 express generalisations, functions and equations algebraically using one or more variables

D2 manipulate algebraic expressions to generate more convenient forms

D3 identify and express recursion and periodicity in various contexts

D4 recognise and determine important features of families of functions

D5 recognise different situations which can be modelled by the same function and fit curves to data sets

D6 formulate equations and systems of equations and solve to appropriate levels of accuracy, making use of graphical, computational and analytical methods

D7 determine feasible regions under sets of constraints

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